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Reg. No. : ....

Name : ....

First Semester B.Sc. Degree Examination, June 2022

First Degree Programme under CBCSS

Mathematics

Complementary Course I for Statistics

MM 1131.4 : MATHEMATICS I – BASIC CALCULUS FOR STATISTICS

(2018 and 2019 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the first **10** questions are compulsory. They carry **1** mark each.

1. Find the third derivative of the function $f(x)=x^3 \sin x$.

2. Define curvature of a curve.

3. Explain why there is no point c is the interval $(0, \pi)$ for $f(x)=\tan x$ such that $f'(c)=0$, even though $f(0)=f(\pi)=0$.

4. What is the sum of the series $\sum_{k=0}^{\infty}\frac{5}{4^k}$?

5. Find the average value of $f(x)=2x$ over $[0, 4]$.

6. Evaluate $\int_0^2 (2-x)^{-1/2}\,dx$.

7. State comparison test of convergence of series.

P.T.O.

8. Find the stationary points of $f(x) = 3x^5 - 5x^3$.

9. State Mean value theorem.

10. Write the n^{th} derivative of e^{2x}.

(10 × 1 = 10 Marks)

SECTION – II

Answer any **eight** questions from among the questions **11** to **22**. These questions carry **2** marks each.

11. Evaluate $\int x^3 e^{-x^2} dx$.

12. Given that Rolle's theorem holds with $b = 2 + 1/\sqrt{3}$ for the function $f(x) = x^3 - 6x^2 + ax + c$ on $(1, 3)$. Find the values of a and b.

13. Show that the maximum curvature of the catenary $y(x) = a\cosh(x/a)$ is $1/a$.

14. For the function $y(x) = x^2 \exp(-x)$, obtain a simple relationship between y and $\frac{dy}{dx}$.

15. Find the length of the curve $y = x^{3/2}$ from $x = 0$ to $x = 1$.

16. Determine whether the series $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \ldots$ converge absolutely.

17. Show that lowest value taken by the function $3x^4 + 4x^3 - 12x^2 + 6$ is –26.

18. Show that the curve $x^3 + y^3 - 8x - 12y - 16 = 0$ touches the y–axis.

19. Find the position and nature of the stationary points of the function $f(x) = \cos ax$ with $a \neq 0$.

20. Sum the series $S = 1 + \frac{2}{2} + \frac{3}{4} + \frac{4}{8} + \ldots$

21. Evaluate the sum $\sum_{n=1}^{N} \frac{1}{n(n+1)(n+2)}$.

22. Expand $f(x) = \cos x$ as a Taylor series about $x = \pi/2$.

(8 × 2 = 16 Marks)

SECTION – III

Answer any **six** questions from among the questions **23** to **31**. Each question carries **4** marks.

23. Use Leibnitz theorem to find the fourth derivative of $x^2 e^{3x}$.

24. Find the Maclaurin's series for $\ln\left(\frac{1+x}{1-x}\right)$.

25. Evaluate the integral $\int_0^2 (2-x)^{-1/4} dx$.

26. Determine the surface area of the cone generated by the line $y = 2x$ from $x = 0$ to $x = h$ about the x-axis.

27. Determine the range of values of x for which the power sides converges :

    $P(x) = 1 + 2x + 4x^2 + 8x^3 + \dots$

28. What semi-quantities results can be deduced by applying Rolle's Theorem to the following functions $f(x)$, with a and c chosen so that $f(a) = f(c) = 0$?

    (a) $x^2 - 6x + 8$

    (b) $\sin x$.

29. State Cauchy's root test and use it to determine whether the following series converges $\sum_{n=1}^{\infty} \left(\frac{1}{n}\right)^n = 1 + \frac{1}{4} + \frac{1}{27} + \dots$

30. Use Integration by parts to evaluate $\int_0^y x^2 \sin x \, dx$.

31. Evaluate the integral $\int \sin^5 x \, dx$.

(6 × 4 = 24 Marks)

SECTION – IV

Answer any **two** questions from among the questions **32** to **35**. These questions carry **15** marks.

32. (a) Show that the entire length of the curve $x^{2/3}+y^{2/3}=a^{2/3}$ which can be parametrized as $x=a\cos^3\theta$ and $y=b\sin^3\theta$ is $6a$.

(b) Find the volume of the solid that is obtained when the region under the curve $y=\sqrt{x}$ over the interval [1, 4] is revolved about the x–axis.

33. (a) Find the area of surface that is generated by revolving about the portion of the curve $y=x^3$ between $x=0$ and $x=1$ about the x-axis.

(b) Equation in polar coordinates of an ellipse with semi-axes a and b is

$\frac{1}{\rho^2}=\frac{\cos^2\phi}{a^2}+\frac{\sin^2\phi}{b^2}$. Find the area A of the ellipse.

34. (a) Discuss the convergence of Riemann Zeta series $\sum_{n=1}^{\infty}\frac{1}{n^p}$ for $p>1$ and $p\le 1$.

(b) Using Lagrange's mean value theorem, prove that $\frac{c-a}{1+c^2}<\tan^{-1}c-\tan^{-1}a<\frac{c-a}{1+a^2}$.

35. (a) Determine the range of values of z for which the following complex power series converges :

$$P(z)=1-\frac{z}{2}+\frac{z^2}{4}-\frac{z^3}{8}+\ldots$$

(b) Sum the series $S(\theta)=1+\cos\theta+\frac{\cos 2\theta}{2!}+\frac{\cos 3\theta}{3!}+\ldots$

(2 × 15 = 30 Marks)