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N – 4043

Reg. No. : .....

Name : .....

First Semester B.Sc. Degree Examination, June 2022

First Degree Programme under CBCSS

Mathematics

Complementary Course I for Statistics

MM 1131.4 : MATHEMATICS I – DIFFERENTIAL CALCULUS

(2021 Admission)

Time : 3 Hours

Max. Marks : 80

SECTION – I

Answer all the questions.

1. Find  $\lim_{x \rightarrow 0} \frac{|x|}{x}$ .
2. Suppose that  $f$  and  $g$  are continuous functions such that  $f(1)=1$  and  $\lim_{x \rightarrow 1} [f(x) + 3g(x)] = 10$ , then find  $g(1)$ .
3. Find the derivative of  $y = \sqrt{x^2 + 2}$ .
4. Find the intervals on which  $f(x) = x^3 - 4x + 3$  is decreasing.
5. Find the stationary points of the function  $f(x) = x^3 - 3x + 2$ .

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6. Find  $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\cos x}{x - \frac{\pi}{2}}$ .
7. Define Relative Minimum of a function.
8. Find the domain of the function  $f(x, y) = \frac{xy}{x-2}$ .
9. Find  $\frac{\partial f}{\partial y}$  if  $f(x, y) = \sin(xy)$ .
10. Write the local linear approximation of a two variable function  $f(x, y)$  at  $(x_0, y_0)$ .  
(10 × 1 = 10 Marks)

## SECTION – II

Answer any eight questions.

11. Find  $\lim_{x \rightarrow 4^-} \frac{x-2}{(4-x)(x+2)}$ .
12. Find  $\lim_{x \rightarrow \infty} \frac{2(3x-4)}{3x+5}$ .
13. Show that  $|x-1|$  is continuous everywhere.
14. Find  $\frac{dy}{dx}$  if  $x^3 + 3xy = 15 + 2x^2 - y^3$ .
15. Evaluate  $I = \int \frac{e^x}{\sqrt{1-e^{2x}}} dx$ .
16. Explain the first derivative test for checking extremum values of a function.
17. Evaluate  $\lim_{x \rightarrow \frac{\pi}{4}} \frac{(1-\tan x)}{\cos 2x}$ .
18. Define the concavity of a function.
19. Find  $\frac{d^2y}{dx^2}$  if  $x^3 - y^3 = 6$ .

20. Show that the function  $f(x) = \begin{cases} \frac{1}{1+e^x} & x \neq 0 \\ 0 & x = 0 \end{cases}$  is not differentiable at  $x = 0$ .
21. Find the level curves of a function  $f(x, y) = x^2 + 4y^2$ .
22. Find  $\frac{\partial f}{\partial y}$  at  $(1, 1)$ , if  $f(x, y) = xe^{xy^2-1}$ .
23. Find  $\frac{\partial^2 f}{\partial x^2}$  at  $(1, \pi)$ , if  $f(x, y) = x \cos(xy)$ .
24. Let  $f(x, y) = y^2x + 5x^3$ , then find the slope of the surface  $z = f(x, y)$  at the point  $(-1, 1)$  in the  $y$ -direction.
25. Let  $f(x, y) = x^2e^y + x$ , find  $f_{yxx}$  at  $(1, 0)$ .
26. If  $r(x, y, z) = \sqrt{x^2 + y^2 + z^2}$  then find  $\frac{\partial r}{\partial z}$ .

(8 × 2 = 16 Marks)

### SECTION – III

Answer any six questions.

27. Evaluate :

(a)  $\lim_{x \rightarrow \infty} \frac{\sqrt{5+4x^2}}{5+6x}$

(b)  $\lim_{x \rightarrow \infty} \sqrt{x^4 + 9} - x^2$

28. Let  $f(x) = \begin{cases} \frac{1}{x+2}; & x < -2 \\ x^2 - 5; & -2 < x \leq 3 \\ \sqrt{x+13}; & x > 3 \end{cases}$ . Find the following limits

(a)  $\lim_{x \rightarrow -2} f(x)$

(b)  $\lim_{x \rightarrow 0} f(x)$

(c)  $\lim_{x \rightarrow 3} f(x)$ .

29. Find the positions and natures of the stationary points of the function  $f(x) = 2x^3 - 3x^2 - 36x + 2$ .

30. Show that the function  $f(x) = \frac{1}{4}x^3 + 1$  satisfy the hypothesis of the mean value theorem over the interval  $[0, 2]$ , and find all values of  $c$  in the interval  $(0, 2)$  at which the tangent line to the graph of  $f$  is parallel to the secant line joining the points  $(0, f(0))$  and  $(2, f(2))$ .

31. Evaluate  $\lim_{x \rightarrow 0} (1 - \sin^2 x)^{\frac{1}{2x^2}}$ .

32. Find  $\frac{dy}{dx}$  if  $y = \frac{(x-5)^2}{(x^2+1)}$ .

33. Find the greatest and least values of  $f(x) = 3x^4 - 2x^3 - 6x^2 + 6x + 1$  in  $[0, 2]$ .

34. Identify the location of intercepts, relative extrema of the function  $y = x^3 - 3x + 2$ .

35. If  $f(x, y) = \begin{cases} -\frac{xy}{x^2 + y^2} & (x, y) \neq (0, 0) \\ 0 & (x, y) = (0, 0) \end{cases}$  then show that  $f_x(x, y)$  and  $f_y(x, y)$  exist at all points.

36. (a) Define local linear approximation  $L(x, y)$  of  $f(x, y)$  at  $(x_0, y_0)$ .  
 (b) Find  $L(x, y)$  at  $(3, 4)$  of  $f(x, y) = \sqrt{x^2 + y^2}$ .
37. Suppose that  $w = \sqrt{x^2 + y^2 + z^2}$ ,  $x = \cos \theta$ ,  $y = \sin \theta$ ,  $z = \tan \theta$ . Use chain rule to find  $\frac{dw}{d\theta}$  at  $\theta = \frac{\pi}{4}$ .
38. Explain Second derivative test. Find the local extreme value of the function  $f(x, y) = xy - x^2 - y^2 - 2x - 2y + 4$ .

(6 × 4 = 24 Marks)

#### SECTION – IV

Answer any two questions.

39. (a) Use implicit differentiation to find  $\frac{dy}{dx}$  for the Folium of Descartes  $x^3 + y^3 = 3xy$   
 (b) Find an equation for the tangent line to the Folium of Descartes at  $\left(\frac{3}{2}, \frac{3}{2}\right)$ .  
 (c) At what points in the first quadrant is the tangent line to the Folium of Descartes horizontal.
40. (a) Find  $x$  such that  
 (i)  $\log_{10} x = \sqrt{2}$   
 (ii)  $5^x = 7$   
 (iii)  $\ln(x + 1) = 5$ .  
 (b) A space shuttle taking off generates a sound level of 150 dB near the launch-pad. A person exposed to this level of sound would experience severe physical injury. By comparison, a car horn at one meter has a sound level of 110 dB, near the threshold of pain for many people. What is the ratio of sound intensity of a space shuttle take off to that of a car horn?  
 (c) Prove that  $\frac{d}{dx} \ln(x) = \frac{1}{x}$ .

41. (a) State mean value theorem Determine all the numbers  $c$  which satisfy the conclusion of mean value theorem for the function  $f(x) = x^3 + 2x^2 - x$  on  $[-1, 2]$ .

(b) Verify Roll's theorem for the function  $f(x) = x^2 + 2x - 8$  in  $[-4, 2]$ .

42. (a) Evaluate  $\lim_{x \rightarrow 0} (\cot x)^{1/\log x}$ .

(b) Calculate  $\lim_{x \rightarrow 0^+} \left[ \frac{1}{x} - \frac{1}{\sin x} \right]$ .

(c) Find  $\lim_{x \rightarrow 0} [1 + \sin x]^{\frac{1}{x}}$ .

43. (a) (i) If  $f(x, y) = xy + x^2 - 4$  find

(ii)  $f(x + y, x - y)$  and

(iii)  $f(xy, 3x^2y^3)$ .

(b) If  $f(x, y) = xy^2 + \cos(xy)$  then find all second order partial derivatives of  $f(x, y)$

(c)  $f(x, y, z) = 2x^2 + 3y^2 + 3z^2$  then find  $f_{xx}(x, y) + f_{yy}(x, y) + f_{zz}(x, y)$  at  $(1, 1, 1)$ .

44. (a) Find the point  $p(x, y, z)$  closest to origin on the plane  $2x + y - z = 0$ .

(b) Find the greatest and smallest values of the function  $f(x, y) = xy$  takes on ellipse  $\frac{x^2}{8} + \frac{y^2}{2} = 1$ .

(2 × 15 = 30 Marks)