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Reg. No. : ....

Name : ....

First Semester B.Sc. Degree Examination, June 2022

First Degree Programme Under CBCSS

Mathematics

Complementary Course I for Statistics

MM 1131.4 : MATHEMATICS I – BASIC CALCULUS FOR STATISTICS

(2020 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – I

(All the **first ten** questions are compulsory. They carry **1** mark each)

1. Find the third derivative of the function $f(x) = x^3 \sin x$.

2. Define curvature of a curve.

3. Explain why there is no point c in the interval $(0,\pi)$ for $f(x) = \tan x$ such that $f'(c) = 0$, even though $f(0) = f(\pi) = 0$.

4. What is the sum of the series $\sum_{k=0}^{\infty} \frac{5}{4^k}$?

5. Find the average value of $f(x) = 2x$ over $[0,4]$.

6. Evaluate $\int_0^2 (2-x)^{-1/2} dx$.

7. State comparison test.

P.T.O.

8. Find the stationary points of $f(x) = 3x^5 - 5x^3$.

9. State Mean value theorem

10. Write the n^{th} derivative of e^{2x}.

(10 × 1 = 10 Marks)

SECTION – II

(Answer **any eight** from among the questions 11 to 26. These questions carry **2** marks each)

11. Evaluate $\int x^3 e^{-x^2} dx$.

12. Evaluate the integral $\int_0^\infty \frac{x}{(x^2 + a^2)^2} dx$.

13. Given that Rolle's Theorem holds with $b = 2 + 1/\sqrt{3}$ for the function $f(x) = x^3 - 6x^2 + ax + c$ on $(1,3)$. Find the values of a and b.

14. Find 'b'of the Mean Value Theorem when $f(x) = x(x-1)$ in $(1,2)$.

15. Show that the maximum curvature of the centenary $y(x) = a\cosh(x/a)$ is $1/a$.

16. For the function $y(x) = x^2 \exp(-x)$, obtain a simple relationship between y and $\frac{dy}{dx}$.

17. Find the length of the curve $y = x^{3/2}$ from $x = 0$ to $x = 1$.

18. State the alternating series test.

19. Determine whether the series $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + ...$ converge absolutely.

20. Show that lowest value taken by the function $3x^4+4x^3-12x^2+6$ is -26.

21. Show that the curve $x^3+y^3-8x-12y-16=0$ touches the y-axis.

22. Find the position and nature of the stationary points of the function $f(x)=\cos ax$ with a $\neq 0$.

23. Sum the series $S=1+\frac{2}{2}+\frac{3}{4}+\frac{4}{8}+\ldots$

24. Evaluate the sum $\sum_{n=1}^{N}\frac{1}{n(n+1)(n+2)}$.

25. Expand $f(x)=\cos x$ as a Taylor series about $x=\pi/2$.

26. Find the inflexion points of $f(x)=x^4-2x^3+2x-1$.

(8 × 2 = 16 Marks)

SECTION – III

Answer **any six** questions from among the question 27 t o38. These questions carry **4** marks.

27. Use Leibnitz theorem to find the fourth derivative of x^2e^{3x}.

28. Determine inequalities satisfied by lnx and sin x for suitable ranges of the real variable x.

29. Find the Maclaurin's series for $n\left(\frac{1+x}{1-x}\right)$.

30. Evaluate the integral $\int_{2}^{0}(2-x)^{-1/4}dx$.

31. Determine the surface area of the cone generated by the line $y=2x$ from $x=0$ to $x=2$ about the x - axis.

32. Determine the range of values of x for which the power series converges:

$P(x)=1+2x+4x^2+8x^3+\ldots$

33. What semi-quantities results can be deduced by applying Rolle's Theorem to the following functions $f(x)$, with a and c chosen so that $f(a)=f(c)=0$?

(a) x^2-6x+8

(b) $\sin x$

34. Given that the series $\sum_{n=1}^{\infty}\frac{1}{n}$ diverges, determine whether the following series converges $\sum_{n=1}^{\infty}\frac{4n^2-n-3}{n^3+2n}$.

35. State Cauchy's root test and use it to determine whether the following series converges:

$$\sum_{n=1}^{\infty}\left(\frac{1}{n}\right)^n=1+\frac{1}{4}+\frac{1}{27}+\ldots$$

36. Use integration by parts to evaluate $\int_0^y x^2\sin x\,dx$.

37. Evaluate the integral $\int\sin^5 x\,dx$.

38. Find the volume of a cone enclosed by the surface formed by rotating about the x-axis, the line $y=2x$ between $x=0$ and $x=h$.

(6 × 4 = 24 Marks)

SECTION – IV

(Answer **any two** questions from among the questions 39 to 44. These questions carry **15** marks each)

39. (a) Show that the entire length of the curve $x^{2/3}+y^{2/3}=a^{2/3}$ which can be parametrized as $x=a\cos^3\theta$ and is 6a.

(b) Find the volume of the solid that is obtained when the region under the curve $y=\sqrt{x}$ over the interval $[1,4]$ is resolved about the x-axis.

40. (a) Find the area of surface that is generated by revolving about the portion of the curve $y = x^3$ between $x = 0$ and $x = 1$ about the x-axis.

(b) Equation in polar coordinates of an ellipse with semi-axes a and b is

$\frac{1}{\rho^2} = \frac{\cos^2\phi}{a^2} + \frac{\sin^2\phi}{b^2}$. Find the area A of the ellipse.

41. (a) Show that the radius of curvature at the point (x, y) on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ has magnitude $\frac{a^4y^2 + b^4x^2}{a^4b^4}$ and the opposite sign to y. Check the special case $b = a$, for which the ellipse becomes a circle.

(b) Show that the value of the integral $I = \int_0^1 \frac{1}{\left(1 + x^2 + x^3\right)^{1/2}} dx$ lies between 0.810 and 0.882.

42. (a) Show that the series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ for $p > 1$ and $p \leq 1$.

(b) Using Lagrange's mean value theorm, prove that

$\frac{c-a}{1+c^2} < \tan^{-1} c - \tan^{-1} a < \frac{c-a}{1+a^2}$.

43. (a) Show that the series $\sum_{n=1}^{\infty} \frac{\cos n}{n^2}$ converges.

(b) Sum the series $S(x) = \frac{x^4}{3(0!)} + \frac{x^5}{4(1!)} + \frac{x^6}{5(2!)} + \ldots$

44. (a) Determine the range of values of z for which the Following complex power series converges:

$$P(z) = 1 - \frac{z}{2} + \frac{z^2}{4} - \frac{z^3}{8} + \ldots$$

(b) Sum the series $S(\theta) = 1\cos\theta + \frac{\cos 2\theta}{2!} + \frac{\cos 3\theta}{3!} + \ldots$

(2 × 15 = 30 Marks)