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Reg. No. : ....

Name : ....

Fourth Semester B.Sc. Degree Examination, July 2018
First Degree Programme under CBCSS
COMPLEMENTARY COURSE FOR CHEMISTRY/POLYMER CHEMISTRY
MM 1431.2 : Mathematics – IV (Abstract Algebra and Linear Transformations)
(2014 Admission Onwards)

Time : 3 Hours Max. Marks : 80

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SECTION – I

All the **first 10** questions are **compulsory**. They carry **1** mark **each**.

1. Find the order of the group $GL_3(Z_2)$.
2. Find the number of elements of order 2 in $Z_2 \times D_4$.
3. Find the order of $\sigma = (4, 5)(2, 3, 7)$ and $\tau = (1, 4)(3, 5, 7, 8)$.
4. Check whether Q_4 and D_4 are isomorphic or not.
5. Show that every group G with O(G) = 65 is cyclic.
6. Find the characteristic of Z_4 and Z_7.
7. Give an example of a finite non commutative ring with unity.
8. Check whether $Z_3[i]$ is a field or not.
9. Check whether the vectors $\begin{bmatrix} 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ -1 \end{bmatrix}, \begin{bmatrix} -2 \\ 2 \end{bmatrix}$ are linearly independent or not.
10. Consider $T : R^3 \to R$ by T (a, b, c) = 3a – 4b + 6c. Check whether T is a linear transformation or not.

P.T.O.

SECTION – II

Answer **any 8** questions from among the questions **11** to **22**. These questions carry **2** marks **each**.

11. Consider the permutations σ and μ in S_6 :

$$\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 4 & 5 & 6 & 2 \end{pmatrix}, \mu = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 2 & 4 & 3 & 1 & 6 \end{pmatrix}$$

Compute $\sigma^2\mu$.

12. Find the number of cyclic subgroups of order 4 in $Z_4 \times Z_4$.

13. Check whether U(20) is cyclic or not.

14. Find a subgroup of $Z_{12} \oplus Z_4 \oplus Z_{15}$ with order 9.

15. How many elements of $Aut(Z_{20})$ are of order 2 ?

16. State the number of automorphisms of the following groups :

Z_6, Z_8 and Z_{17}.

17. Find the order of the element (4, 9) in $Z_{10} \times U(10)$?

18. Write all the units in each of the following rings.

a) Z b) $Z \times Q \times Z$ c) Z_4.

19. Find the characteristic of each of the following rings :

a) $Z \times Z$ b) $Z_3 \times Z_3$ c) $Z_6 \times Z_{15}$

20. Let $\{v_1, v_2, v_3\}$ be a linearly independent set of vectors in $V_3(R)$. Show that $\{2v_1 + v_2, v_3 + v_2, v_1 - v_3\}$ is linearly independent.

21. Determine whether the set of vectors are linearly independent or not in $V_3(R)$: {(1, 2, 3), (2, 3, 1)}.

22. Prove that $T : R^3 \to R^3$ defined by T(a, b, c) = (a, b, 0) is a linear transformation.

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SECTION – III

Answer **any 6** questions from among the questions **23** to **31**. These questions carry **4** marks **each**.

23. Define $*$ on Q^+ by $a * b = ab/2$. Show that $(Q^+, *)$ is a group.

24. Construct the Cayley table for D_3.

25. In the group G, if $a^5 = e$ and $aba^{-1} = b^2$, for some $a, b \in G$, find O(b).

26. Find the number of elements of order 3 in A_5.

27. Find the number of subgroups in D_4 which are not cyclic.

28. Find all subgroups of Q_4.

29. Assuming that $\begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}^n = \begin{bmatrix} \cos n\theta & -\sin n\theta \\ \sin n\theta & \cos n\theta \end{bmatrix}$ for any positive integer n and any angle θ in the group SL(2, R). Find the order of $\begin{bmatrix} \cos 60 & -\sin 60 \\ \sin 60 & \cos 60 \end{bmatrix}$

30. Show that 2ZU5Z is not a subring of Z.

31. Prove that $T : R^3 \to R$ defined by $T(x, y, z) = x^2 + y^2 + z^2$ is not a linear transformation.

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SECTION – IV

Answer **any 2** questions from among the questions **32** to **35**. These questions carry **15** marks **each**.

32. i) The group $S_3 \oplus Z_2$ is isomorphic to one of the groups Z_{12}, $Z_6 \oplus Z_2$, A_4, D_6. Pick out the group.

ii) The group $Z_2 \oplus D_3$ is isomorphic to one of the groups Z_{12}, $Z_2 \oplus Z_2 \oplus Z_3$, A_4, D_6. Pick out the group.

33. i) Find all solutions of the equation $x^3 - 2x^2 - 3x = 0$ in Z_{12}.

ii) Find all solutions of the equation $x^2 + 2x + 2 = 0$ in Z_6.

34. Let $v_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, $v_2 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $v_3 = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$.

a) Determine if the set $\{v_1, v_2, v_3\}$ is linearly independent or not.

b) If possible, find a linearly dependent relation among v_1, v_2 and v_3.

35. Let $T(x, y, z) = (2x, 4x - y, 2x + 3y - z)$. Show that T is a linear transformation. Also verify T is invertible or not.

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