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Reg. No. : ....

Name : ....

First Semester M.Sc. Degree Examination, August 2021

Physics

PH 212 : MATHEMATICAL PHYSICS

(2018 – 2019 Admission)

Time : 3 Hours Max. Marks : 75

PART – A

I. Answer any **five** questions. Each question carries **3** marks.

(a) Represent Cartesian unit vectors in spherical polar unit vectors.

(b) Write a short note on the method of discrete Fourier transform.

(c) Write down the characteristics of the normal distribution.

(d) Show the recurrence relation for the Legendre polynomials $P_n(x)$.

$(2n+1)xP_n(x) = (n+1)P_{n+1}(x) + nP_{n-1}(x)$.

(e) S.T. $\frac{d}{dx}\left[x^{-n}J_n(x)\right] = -x^{-n}J_{n+1}(x)$, where $J_n(x)$ is the Bessel function of first kind.

(f) Distinguish between regular and irregular representations.

(g) What is contraction? S.T. contraction of a tensor reduces its rank by two.

(h) Show that the identity element in a group is unique.

(5 × 3 = 15 Marks)

P.T.O.

PART – B

Answer A or B part of questions from II to IV. Each question carries **15** marks.

II. (A) (a) What are scale factors? Obtain scale factors in spherical polar co-ordinate system. **5**

(b) Obtain the Fourier series for the Saw-Tooth wave $f(x) = \begin{cases} x, & 0 \le x < \pi \\ x - 2\pi, & \pi \le x < 2\pi \end{cases}$. **10**

OR

(B) (a) Obtain an expression for the derivative of $f(z)$ of a complex variable z, and hence show that if $f(z)$ is analytic, so its derivative. **10**

(b) Obtain an expression for the mean of the Binomial distribution. **5**

III. (A) (a) Show that a second order homogeneous ordinary differential equation has two linearly independent solutions. **7**

(b) Using the method of Wronskian, show that the functions $\left\{1, \frac{x^n}{n!}\right\}$ $(where\ n = 1, 2, \ldots n-1)$, are linearly independent of each other. **8**

OR

(B) (a) Obtain the Green's function corresponding to the differential equation $y'' - y = f(x),\ y(\pm\infty) = 0$. **9**

(b) Obtain the eigen function expansions of Green's function. **6**

IV. (A) (a) State and prove Schur's Lemma's 1 and 2. **10**

(b) From the Schur's lemmas obtain the orthogonality theorem. **5**

OR

(B) (a) Show that the covariant derivative of fundamental tensors $g_{\mu,\upsilon}$, $g^{\mu\upsilon}$ and g^{μ}_{υ} are all identically zero. **10**

(b) S.T. if all components of a tensor of any rank vanish in one co-ordinate system, they vanish in all coordinate systems. **5**

(3 × 15 = 45 Marks)

PART – C

V. Answer any **three** questions. Each question carries **5** marks.

(a) Obtain Taylor series expansion of $\ln(1+z)$.

(b) If z_0 is a pole of order m of $f(z)$, show that the residue $a_{-1} = \frac{1}{(m-1)!}\left[\frac{d^{m-1}}{dz^{m-1}}\{f(z)(z-z_0)\}^m\right]_{z=z_0}$.

(c) Assuming that on an average, one telephone out of 10 is busy. Six telephone number are randomly selected and called. Find the probability that 4 out of them would be busy.

(d) If $L\{f(t)\} = f(s)$, then S.T. $L\{F(at)\} = \frac{1}{a} f\left({s}/{a}\right)$ where L is the Laplace transform operator.

(e) Obtain Rodrigues representation of Legendre polynomial.

(f) State and explain the quotient rule for tensors.

(3 × 5 = 15 Marks)