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N – 7763

Reg. No. :

Name :

Fourth Semester B.Sc. Degree Examination, August 2022

First Degree Programme under CBCSS

Mathematics

Complementary Course for Physics

**MM 1431.1 : MATHEMATICS IV – COMPLEX ANALYSIS, SPECIAL
FUNCTIONS AND PROBABILITY THEORY**

(2018 Admission)

Time : 3 Hours

Max. Marks : 80

SECTION – I

Answer all questions. Each carries 1 mark.

1. Find the real and imaginary part of e^z .
2. Define an analytic function.
3. Evaluate $\int_1^{1+i} z \, dz$ along straight line parallel to x-axis.
4. Find the residue of $\frac{\cos z}{z}$ at $z = 0$.
5. Find $\Gamma(10)/\Gamma(8)$.
6. Define $\beta(p, q)$.

P.T.O.

7. In three tosses of a coin, find the probability that all three are heads?
8. What is probability of getting a white ball from a box containing 3 white balls and 2 red balls?
9. Out of 50 members of a club in how many ways can a president, vice president and a secretary can be chosen.
10. A number is selected from numbers 1 to 11 at random. What is the probability of choosing an odd number?

(10 × 1 = 10 Marks)

SECTION – II

Answer any **eight** questions. Each carries **2** marks.

11. Check whether the function $\frac{y - ix}{x^2 + y^2}$ is analytic or not.
12. Using polar co-ordinate find out whether $f(z) = \sqrt{z}$ satisfy Cauchy-Riemann equation.
13. Evaluate $\int_C \frac{\sin z \, dz}{2z - \pi}$ where C is the circle $|z| = 2$.
14. Evaluate $\int_C \frac{e^{2z} \, dz}{(z - 2)^2}$ where C is the circle $|z| = 3$.
15. Define (a) Pole (b) Essential singularity.
16. Find the residue of $f(z) = \frac{z}{(2z + 1)(5 - z)}$ at $z = 5$.
17. Find $\Gamma\left(\frac{1}{4}\right) \div \Gamma\left(\frac{9}{4}\right)$.
18. Express $\int_0^\infty x^{-2/3} e^{-x} \, dx$ as Γ function.

19. Find the probability of drawing an ace or a spade from a pack of cards.
20. One bag contains 5 red and 3 white balls. A second contains 4 red and 7 black balls. If one ball is drawn at random from each bag, what is the probability that both are of the same colour?
21. If a die is rolled three times, what is the probability of getting 5 atleast once?
22. Distinguish between discrete and continuous probability functions.

(8 × 2 = 16 Marks)

SECTION – III

Answer any six questions. Each carries 4 marks.

23. Derive Cauchy-Reimann equation.
24. Show that the function $x^2 - y^2$ is harmonic. Find the function $f(z)$ for which the given function is the real part.
25. Expand $\frac{1}{z^2(1+z)^2}$ as Laurent series in the region (i) $0 < |z| < 1$ (ii) $|z| > 1$.
26. Solve $\int_0^{\infty} x^5 e^{-x^2} dx$.
27. Evaluate $\Gamma\left(\frac{1}{2}\right)$.
28. A committee of 5 persons is to be selected randomly from a group of 5 men and 10 women. Find the probability that the committee consists of 2 men and 3 women.
29. A player is to toss three coins. He wins Rs. 10 if three heads appear, Rs. 5 if two heads appear, Rs. 1 if one head appears. He will lose Rs. 12 if no head appears. What is the expected amount?

30. If 3% of electric bulbs manufactured by a company are defective, find the probability that in a sample of 100 bulbs exactly five bulbs are defective.
31. Find the mean and variance of a random variable x which takes values 0, 1, 2, 3 with respective probabilities $\frac{1}{8}, \frac{3}{8}, \frac{3}{8}, \frac{1}{8}$.

(6 × 4 = 24 Marks)

SECTION – IV

Answer any **two** questions. Each carries **15** marks.

32. (a) Evaluate using Residue theorem $\int_0^{2\pi} \frac{d\theta}{5 - 3\cos\theta}$.
- (b) Find the residue of $f(z) = \frac{z \sin z}{(z - \pi)^3}$ at $z = \pi$.
33. (a) Prove that $\beta(p, q) = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}$.
- (b) Find $\int_0^{\infty} \frac{y^2 dy}{(1+y)^6}$.
34. (a) Find the probability of exactly 52 heads in 100 tosses of a coin using the binomial distribution and using normal approximation.
- (b) Find the probability $P(45, 55)$ of between 45 and 55 heads in 100 tosses of a coin. That is $45 \leq x \leq 55$.
35. Eight unbiased coins were tossed simultaneously. Find the probability of getting
 (a) exactly 4 heads (b) no heads at all (c) 6 or more heads (d) at most two heads
 (e) number of heads ranging from 3 to 5.

(2 × 15 = 30 Marks)