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Reg. No. : ....

Name : ....

Second Semester M.Sc. Degree Examination, July 2019

Branch : Mathematics

MM 222 — REAL ANALYSIS – II

(2017 Admission Onwards)

Time : 3 Hours Max. Marks : 75

Instruction :

Answer **one** question from each unit. Each question carries **15** marks.

UNIT I

I. (a) Prove that the outer measure of an interval is its length. 9

(b) If $\{E_j\}_{j=1}^{\infty}$ is a sequence of measurable sets, then prove that $\bigcup_{j=1}^{\infty} E_j$ is measurable. 6

II. (a) Let C be any real number and let f and g be real valued measurable functions defined on the same measurable set E. Then prove that $f+c$, cf, $f+g$, $f-g$ and fg are measurable. 9

(b) Prove that not every measurable set is a Borel set. 6

UNIT II

III. (a) State and prove Fatou's Lemma. 9

(b) Let f and g be non-negative measurable functions. Prove that $\int f+g\ dx = \int f\ dx + \int g\ dx$. 6

P.T.O.

IV. (a) State and prove Lebesgue dominated convergence theorem. **8**

(b) If f is Riemann integrable and bounded over the finite interval $[a, b]$, then prove that f is integrable and $R\int_a^b f\,dx = \int_a^b f\,dx$. **7**

UNIT III

V. (a) Let μ^* be an outer measure on $H(R)$ and let S^* denote the class of μ^*–measurable sets. Then prove that S^* is a σ–ring and μ^* restricted to S^* is a complete measure. **8**

(b) If μ is a σ–finite measure on a ring R, then prove that it has a unique extension to the σ–ring $S(R)$. **7**

VI. (a) Let $[[X, S, \mu]]$ be a measure space and f a non-negative measurable function. Then prove that $\phi(E) = \int_E f\,d\mu$ is a measure on the measurable space $[[X, S]]$. If, in addition, $\int_E f\,d\mu < \infty$, then prove that $\forall \in > 0$, $\exists \delta > 0 E$ such that, if $A \in S$ and $\mu(A) < \delta$, then $\phi(A) < \in$. **8**

(b) Let $\{f_n\}$ be a sequence of integrable functions such that $\sum_{n=1}^{\infty}\int |f_n|\,d\mu < \infty$. Then prove that $\sum_{n=1}^{\infty} f_n$ converges a.e. its sum f, is integrable and $\int f\,d\mu = \sum_{n=1}^{\infty}\int f_n\,d\mu$. **7**

UNIT IV

VII. (a) Prove that differentiable function ψ is convex on (a, b) if and only if ψ' is a monotone increasing function. If ψ'' exists on (a, b) then prove that ψ is convex if and only if $\psi'' \geq 0$ on (a, b). **8**

(b) State and prove Jensen's inequality. **7**

VIII. (a) If $1 \le p < \infty$ and $\{f_n\}$ is a sequence in $L^p(\mu)$ such that $\|f_n - f_m\| \to 0$ as $m, n \to \infty$, then prove that there exist a function f and a subsequence $\{n_i\}$ such that $\lim_{n \to \infty} f_{n_i} = f$ a.e. Also prove that $f \in L^p(\mu)$ and $\lim \|f_n - f\|_p = 0$. **8**

(b) Let $\{f_n\}$ be a sequence in $L^\infty(\mu)$ such that $\|f_n - f_m\|_\infty \to 0$ as $n, m \to \infty$. Then prove that there exist a function f such that $\lim f_n = f$ a.e., $f \in L^\infty(\mu)$ and $\lim \|f_n - f\|_\infty = 0$. **7**

UNIT V

IX. (a) Let γ be a signed measure on $[[X, S]]$. Let $E \in S$ and $\gamma(E) > 0$. Then prove that there exists a positive set A with respect to μ such that $A \subseteq E$ and $\gamma(A) > 0$. **8**

(b) Let γ be a signed measure on $[[X, S]]$. Then prove that there exist measures γ^+ and γ^- on $[[X, S]]$ such that $\gamma = \gamma^+ - \gamma^-$ and $\gamma^+ \perp \gamma^-$. **7**

X. State and prove Radon-Nikodym theorem. **15**

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