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P – 1222

Reg. No. : ....

Name : ....

Second Semester B.A. Degree Examination, September 2022

First Degree Programme under CBCSS

Mathematics

Complementary Course for Economics

MM 1231.5 : MATHEMATICS FOR ECONOMICS II

(2020 Admission)

Time : 3 Hours

Max. Marks : 80

SECTION – I

All the first **ten** questions are compulsory. They carry **1** mark each.

1. Write the derivative of $\tan x$.

2. State quotient rule of differentiation.

3. Find $\frac{d}{dx}\left(2x^3 + x^2\right)$.

4. If total cost is $x^2 + 2x + 3$, find marginal cost.

5. When do we say that a function is differentiable at a point?

6. State product rule of partial differentiation to find $\frac{\partial z}{\partial x}$, where $z = g(x, y).h(x, y)$.

7. Define inflection point.

8. What is a critical point?

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9. Find second derivative of $y = e^x$.

10. What are homogeneous functions?

(10 × 1 = 10 Marks)

SECTION – II

Answer any **eight** questions. These questions carry **2** marks each.

11. If the linear demand function $Q = -8P + 425$, find the total revenue function for the producer.

12. If total cost function $TC = 2Q^2 + 3Q + 52$ and total revenue function $TR = -Q^2 + 55Q$, find the marginal cost and marginal revenue.

13. Show that the marginal revenue can be expressed as $p + x\frac{dp}{dx}$.

14. Find the derivative of $y = \frac{x^2 + 1}{x^3}$.

15. Find the critical points of $f(x) = 3x^3 - 54x^2 + 288x - 22$.

16. Find $\frac{dy}{dx}$ where $x = \sin t$ and $y = \tan t$.

17. Plot the graph of the function $y = 4x - x^2$.

18. If x and y satisfy the equation $x^2 + y^2 = 1$, prove that $\frac{dy}{dx} = -\frac{x}{y}$.

19. Find the critical point of $f(x) = x^2 + 2x + 1$.

20. Differentiate $y = (x^2 + 1).\log(x^2 + 1)$.

21. Find the partial derivatives of z with respect to x and with respect to y, where $z = e^{4x^2y^3}$.

22. Find dy/dx for the implicit function $9x^2 - y = 0$.

23. Find z_x at the point $x = 1$, $y = 2$, where $z = 2x^3y^4$.

24. Find the conditions for a function $f(x, y)$ to have relative minimum.

25. Find dy/dx where $2x^3 - 3y^2 = 96$.

26. If $z = \log(e^x + e^y)$, prove that $\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 1$.

(8 × 2 = 16 Marks)

SECTION – III

Answer any **six** questions. These questions carry **4** marks each.

27. Find equilibrium price and equilibrium quantity given Supply: $Q = 5P + 10$ and Demand: $Q = -3P + 50$, using (a) equations (b) graphs

28. If total cost and total revenue functions are $TC = 2Q^3 - Q^2 + 80Q + 150$ and $TR = 800Q - 7Q^2$, find maximum profit.

29. If $f(x) = 5x^4 + 8x^3 + 7x^2$, find $f'(x), f''(x), f'''(x)$ and $f^{(4)}(x)$.

30. Given $z = 4x^5 + 7xy + 8y^4$, prove that $z_{xy} = z_{yx}$.

31. Given $z = (8x + 15y)(12x - 7y)$, use product rule of differentiation find z_x and z_y.

32. Find both first order partial derivatives of $z = \ln|4x + 9y|$.

33. Find z_x and z_y at the point $x = 1, y = 2$, where $z = 2x^3y^4$.

34. Using power rule, find the fourth order derivative of $y = (x + 4)^3$.

35. If $x = r\cos\theta, y = r\sin\theta$, prove that $\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial y^2} = 0$.

36. Write a short note on geometrical interpretation of partial derivatives.

37. Give any two applications of derivatives in business and economics.

38. A firm's short run production function is given by $Q = 6L^2 - 0.2L^3$ where L denotes the number of workers. Find the size of the workforce that maximizes output and hence sketch a graph of this production function.

(6 × 4 = 24 Marks)

SECTION – IV

Answer any **two** questions. These questions carry **15** marks each.

39. Given that total revenue $R(x) = 600x - 5x^2$ and total cost $C(x) = 100x + 10500$.

    (a) Express profit as a function of x

    (b) Determine maximum level of profit

    (c) Sketch the graph of profit function

40. If $u = \log\sqrt{x^2 + y^2}$, prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$.

41. Differentiate the following equations

    (a) $\frac{e^x \log x}{x^2}$

    (b) $\sqrt{\frac{1+x}{1-2x}}$

    (c) $x^3 + y^3 + 3xy = 0$

42. Find the critical points and test to see if the function is at relative maximum or minimum given $z = 5x^2 - 8x - 2xy - 6y + 4y^2 + 27$.

43. Use the Lagrange multiplier method to optimize the function $z = 6x^2 + 5xy + 2y^2$ subject to the constraint $2x + y = 96$.

44. (a) Determine the first and second order partial derivatives of $f(x, y) = x^3y^2$.

    (b) State Euler's theorem for homogeneous function and verify it for $z = x^2 + 3xy + 2y^2$.

(2 × 15 = 30 Marks)