

Reg. No. :

Name :

Second Semester B.Sc. Degree Examination, September 2022

First Degree Programme under CBCSS

Mathematics

Foundation Course II

MM 1221 : FOUNDATIONS OF MATHEMATICS

(2014 – 2017 Admission)

Time : 3 Hours

Max. Marks : 80

SECTION - I

Answer all questions. Each question carries 1 mark. Answer in one word to a maximum of two sentences.

1. Find the order of 2 modulo 5.
2. Find the inverse of [2] in $\mathbb{Z}/5\mathbb{Z}$.
3. In which congruence class in $\mathbb{Z}/3\mathbb{Z}$ is 2015.
4. Find $\phi(437)$.
5. Find the x-coordinates of all inflection points of $x^4 - 5x^3 + 9x^2$.
6. Evaluate $\int \frac{\sin x}{\cos^2 x} dx$.

P.T.O.

7. Find $\int_{-2}^3 f(x) dx$ if $\int_{-2}^1 f(x) dx = 2$ and $\int_1^3 f(x) dx = -6$.
8. Find the average value of $f(x) = x^2$ over $[0, 1]$.
9. Find the rectangular co-ordinates of the point whose polar co-ordinates are $(4, \pi/6)$.
10. Find the eccentricity and the distance from the pole to the directrix of the conic $r = \frac{3}{2 - 2\cos\theta}$.

(10 × 1 = 10 Marks)

SECTION – II

Answer any **eight** questions. Each question carries **2** marks.

11. In $\mathbb{Z}/17\mathbb{Z}$, solve $[3]X - [11]$.
12. In $\mathbb{Z}/12\mathbb{Z}$, find the inverses of $[1]$, $[5]$, $[7]$ and $[11]$.
13. Find whether $\{1, 4, 9, 16, 25, 36, 49\}$ is a complete set of representatives for $\mathbb{Z}/17\mathbb{Z}$.
14. If n is prime, prove that $\phi(n) = n - 1$.
15. Write down the table for addition modulo 7.
16. Find remainder when 2^{999} is divided by 13.
17. Evaluate $\int_0^3 f(3x) dx$ if $\int_0^9 f(x) dx = 5$.
18. Evaluate $\int_{-1}^1 x^2 |x| dx$.

19. Find the displacement and distance travelled during the time interval $0 \leq t \leq 3$ if the velocity function is $v(t) = 3t^2 + 2t$.
20. Evaluate $\int_{\pi/4}^{\pi/2} \cos \theta (1 - 9 \sin^2 \theta) d\theta$.
21. Find the length of the curve $y = 2x$ from $(1, 2)$ and $(2, 4)$.
22. Find the entire area within the cardioids $r = 1 - \cos \theta$.

(8 × 2 = 16 Marks)

SECTION – III

Answer any **six** questions. Each question carries **4** marks.

23. Find the orders of the non zero elements of $\mathbb{Z}/7\mathbb{Z}$.
24. In $\mathbb{Z}/m\mathbb{Z}$, show that $[a]$ is a unit if and only if $(a, m) = 1$.
25. State Fermat's theorem and verify it for $a = 3$ and $p = 7$.
26. Find the surface area of the solid generated by revolving the curve $y^2 = 8x$ about the x -axis from $x = 2$ to $x = 7$.
27. Evaluate without finding anti-derivative.

(a) $\int_0^4 2 dx$

(b) $\int_0^1 \sqrt{1-x^2} dx$

28. At each point (x, y) on the curve, y satisfies the condition $\frac{d^2y}{dx^2} = 6x$; the line $y = 5 - 3x$ is a tangent to the curve at the point where $x = 1$. Find the equation of the curve.

29. Find the volume of the solid generated by revolving the part of the curve $x^2(y - x^2) = 3$ between $x = 1$ and $x = 2$ about the x -axis.
30. Find the surface area generated by revolving one arch of the cycloid $x = a(\theta + \sin \theta)$, $y = a(1 + \cos \theta)$ about the x -axis.
31. Find the relative extreme of $f(x, y) = 2xy - x^3 - y^2$.

(6 × 4 = 24 Marks)

SECTION – IV

Answer any two questions. Each question carries 15 marks.

32. (a) State and prove Fermat's theorem and verify it for $a = 3$ and $p = 7$.
- (b) If e is the order of a modulo m and $a^e \equiv 1 \pmod{m}$, then e divides $\phi(m)$.
33. (a) If $2^{41} \equiv 1 \pmod{83}$, find the order of $2 \pmod{83}$.
- (b) Prove that $n^9 + 2n^7 + 3n^4 + 4n$ is divisible by 5 for any integer n .
34. (a) Use cylindrical shells to find the volume of the solid generated when the region enclosed between $y = \sqrt{x}$, $x = 1$, $x = 4$ and the x -axis is revolved about the y -axis.
- (b) Find the length of the curve $24xy = y^4 + 48$ from $y = 2$ to $y = 4$.
35. (a) State the horizontal line test. Use the horizontal line test to check whether the functions (i) $f(x) = |x|$ (ii) $f(x) = \sqrt{x-1}$ and (iii) $f(x) = 3x + 2$ have an inverse by sketching the graph.
- (b) Sketch the graph of $r = 4 \cos \theta$ in polar coordinates.

(2 × 15 = 30 Marks)