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Reg. No. : ....

Name : ....

Second Semester B.Sc. Degree Examination, September 2022

First Degree Programme under CBCSS

Mathematics

Complementary Course for Statistics

MM 1231.4 : MATHEMATICS II – ADVANCED DIFFERENTIAL AND INTEGRAL CALCULUS

(2018 & 2019 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the first **ten** questions are compulsory. Each carries **1** mark.

1. Find $\frac{\delta^2 f}{\delta y\, \delta x}$ if $f(x,y)=2x^3y^2+y^3$

2. Show that $3x\,dy+3y\,dx$ is inexact.

3. Write chain rule for partial differentiation of a function in 2 variables.

4. Evaluate $\int_0^1\int_0^{1-x} x^2y\,dy\,dx$.

5. Write the integral equation for volume under the surface $z=f(x,y)$ above the region R.

P.T.O.

6. Write the Jacobian $J = \frac{\delta(x,y)}{\delta(u,v)}$ as a 2×2 determinant.

7. Find 0! using definite integral.

8. Define gamma function for any $p > 0$.

9. Simplify $\frac{\Gamma(5/4)}{\Gamma(9/4)}$.

10. Evaluate the beta function $B(4,1)$

(10 × 1 = 10 Marks)

SECTION – II

Answer any **eight** questions. Each carries **2** marks.

11. Find the total differential of the function $f(x,y) = y\exp(x+y)$.

12. Show that $(y+z)dx + xdy + xdz$ is exact.

13. Find the rate of change of $f(x,y) = xe^{-y}$ with respect to u if $x(u) = 1 + au$ and $y(u) = bu^3$.

14. Define saddle point of a two variable function.

15. Evaluate $\iint_R x^2 y\, dx\, dy$, where R is the region bounded by the lines $x = 0, y = 0$, and $x + y = 1$.

16. Evaluate $\int_0^1\int_0^1\int_0^1 xyz\, dx\, dy\, dz$

17. Find the volume of the region bounded by the three co-ordinate surfaces $x = 0, y = 0, z = 0$ and the plane $\frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$.

18. Prove that the area of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ is πab.

19. Simplify $\frac{\Gamma(2/3)}{\Gamma(8/3)}$.

20. Prove that $B(p,q)=B(p,q)$.

21. Express beta function in terms of gamma function and hence evaluate $B(4,5)$.

22. Evaluate $\int_0^\infty \frac{x^3}{(1+x)^5}dx$ using beta function.

(8 × 2 = 16 Marks)

SECTION – III

Answer any **six** questions. Each carries **4** marks.

23. Find the Taylor series expansion of $f(x,y)=y \exp xy$ about the point $x=2, y=3$, up to quadratic terms in $x-2$ and $y-3$.

24. Show that $f(x,y)=x^3\exp(-x^2-y^2)$ has a maximum at $\left(\sqrt{\frac{3}{2}},0\right)$ and minimum at $\left(-\sqrt{\frac{3}{2}},0\right)$.

25. Derive the conditions for maxima for a function of two real variables.

26. Find the volume of the region bounded by the paraboloid $z=x^2+y^2$ and the plane $z=2y$.

27. Evaluate $\iint_R \left(a+\sqrt{x^2+y^2}\right)dx\,dy$, where R is the region bounded by the circle $x^2+y^2=a^2$.

28. Evaluate the surface integral of $f(x,y)=(b-y+x)^{-\frac{3}{2}}$ over the rectangle $0 \le x \le a, 0 \le y \le b$.

29. Prove that $\Gamma\left(\frac{1}{2}\right)=\sqrt{\pi}$.

30. Prove that $B(p,q)=\int_0^{\infty}\frac{y^{p-1}}{(1+y)^{p+q}}dy$.

31. Express as beta function and hence evaluate the integral $\int_0^{\frac{\pi}{2}}\sqrt{\sin^3 x \cos x}\, dx$.

(6 × 4 = 24 Marks)

SECTION – IV

Answer any **two** questions. **Each** carries **15** marks.

32. Find the stationary points of $f(x,y,z)=x^3+y^3+z^3$ subject to the constraints. $g(x,y,z)=x^2+y^2+z^2=1$ and $h(x,y,z)=x+y+z=0$.

33. Evaluate the integral $I=\int_{-\infty}^{\infty}e^{-x^2}dx$.

34. Find an expression for a volume element in spherical polar coordinates and hence calculate the moment of inertia about a diameter of a uniform sphere of radius a and mass M.

35. (a) Prove that $B(p,q)=\frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)}$. 10

(b) Evaluate $\int_0^{\frac{\pi}{2}}\frac{d\theta}{\sqrt{\sin\theta}}$. 5

(2 × 15 = 30 Marks)
