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Reg. No. : ....

Name : ....

Second Semester B.Sc. Degree Examination, September 2022

First Degree Programme under CBCSS

Mathematics

Foundation Course – II

MM 1221 : FOUNDATIONS OF MATHEMATICS

(2020 Admission Onwards)

Time : 3 Hours Max. Marks : 80

SECTION – I

Answer **all** questions. Each question carries **1** mark.

Answer in **one** word to a maximum of **two** sentences.

1. Find the range of the function $f(x) = 11 + 5\cos x$.

2. Prove that $p \wedge \sim p$ is a contradiction.

3. Give an example of a relation which is reflexive, symmetric but not transitive.

4. Let $f(x) = \sqrt{x+1} + 4$. The natural domain of f is ____________.

5. Write the negation of the statement : If she works she will earn money.

6. Find the rectangular coordinates of the point P whose polar coordinates are given by $(r, \theta) = \left(6, \frac{2\pi}{3}\right)$.

P.T.O.

7. Identify the curve $r = 4 \sin \theta$ by transforming to rectangular coordinates.

8. State the reflection property of ellipse.

9. Identify the quadratic surface $z = \frac{x^2}{a^2} + \frac{y^2}{b^2}$.

10. Find a normal vector for the plane $4x - 2y + 7z - 11 = 0$.

(10 × 1 = 10 Marks)

SECTION – II

Answer **any eight** questions. Each question carries **2** marks.

11. Determine whether the following sentence is a statement. In 2003 George W. Bush was the president of the United States.

12. Define the terms : Converse and contrapositive.

13. Write the negation of the statement : If x is odd, then $x^2 - 1$ is even.

14. Find the truth value of the implication, if 3 + 3 = 6 then 3 + 4 = 9.

15. Using truth table, show that the statement $q \vee (p \vee \neg q)$ is a tautology.

16. Show that the function $f : R \to R$ defined by $f(x) = 3x + 7$ is one – to – one.

17. Graph the parametric curve $x = 2t - 3$, $y = 6t - 7$ by eliminating the parameter.

18. Find the circumference of a circle of radius a from the parametric equations $x = a\cos t$, $y = a\sin t$ $(0 \le t \le 2\pi)$.

19. Find the arc length of the spiral $r = e^{\theta}$ distance travelled between $\theta = 0$ and $\theta = \pi$.

20. Find the slope of the tangent line to the unit circle $x = \cos t$, $y = \sin t$ $(0 \le t \le 2\pi)$ at the point where $t = \frac{\pi}{6}$.

21. Find the equation of the hyperbola with vertices (0, $\pm$ 8) and asymptotes $y = \pm 4x/3$.

22. Find the asymptotes of the hyperbola $\frac{x^2}{4} - \frac{y^2}{9} = 1$.

23. Find the unit vector that has the same direction as $v = \mathbf{i} + 2\mathbf{j} - 2\mathbf{k}$.

24. Find the new coordinates of the point (2, 4) if the coordinate axes rotated through an angle of 30°.

25. Sketch the graph of $x^2 + y^2 = 1$ in 3-space.

26. Find the direction cosine of the vector $v = 2\mathbf{i} - 4\mathbf{j} - \mathbf{k}$.

(8 × 2 = 16 Marks)

SECTION – III

Answer **any six** questions. Each question carries **4** marks.

27. Construct the truth table for $[(\neg q) \wedge (p \Rightarrow q) \Rightarrow (\neg p)]$.

28. Prove or give a counter example that "for every integer n, $n^2 + 3n + 8$ is even".

29. Negate and simplify the statement $\forall\, x[p(x) \rightarrow q(x)]$.

30. Show that the function $f : \mathbf{R} \rightarrow \mathbf{R}$ defined by $f = \{(x, y);\, y = mx + b\}$ is invertible. Also find its inverse.

31. Let $f : A \rightarrow B$ and $g : B \rightarrow C$ are injective, show that $g \circ f$ is injective.

32. Determine whether the planes

$2x - 8y - 6z - 6 = 0$ and $-x + 4y + 3z + 4 = 0$ are perpendicular.

33. Find the slope of the tangent line to the circle $r = 4\cos\theta$ at the point where $\theta = \frac{\pi}{4}$ and hence show that the circle has a horizontal tangent line at the point.

34. Find the entire area within the cardioid $r = 1 - \cos\theta$.

35. Describe the graph of the equation $y^2 - 8x - 6y - 23 = 0$.

36. (a) Find the vector of length 2 that has an angle of $\frac{\pi}{4}$ with the positive x-axis.

(b) Find the angle that the vector makes with the positive x-axis.

37. Let $A = \{1, 2, 3, 4, 5\}$. Consider the relation R on A defined as $R = \{(2, 2), (4, 4), (5, 5), (2, 5), (5, 2)\ (3, 3)\}$. Is R an equivalence relation?

38. Find the parametric equation of the line

(a) passing through (4, 2) and parallel to $v = (-1, 5)$

(b) passing through (–1, 2, 4) and parallel to $v = 3\mathbf{i} - 4\mathbf{j} + 2\mathbf{k}$.

(6 × 4 = 24 Marks)

SECTION – IV

Answer any **two** questions. Each question carries **15** marks.

39. The relation R on the set of integers Z is defined by xRy if and only if $x - y = 4k$ for some integer k.

(a) Verify that R is an equivalence relation on Z

(b) Determine the equivalent classes and a partition of Z induced by R.

40. (a) Determine the truth value of the following statements with suitable justification :

(i) $\forall x \ \exists y$ such that $x+y=3$

(ii) $\forall x \ \exists y$ such that $x+y\neq 3$

(b) Let $f:A\to B$ and $g:B\to C$ be bijective functions. Show that the composition $g\circ f:A\to C$ is also bijective.

41. (a) Find the equation of the curve $2x^2+xy+2y^2-1=0$ in $x'y'$- coordinate system obtained by rotating the xy-coordinate system through an angle of 45°.

(b) Sketch the graph of $r=\dfrac{2}{1-\cos\theta}$ in polar coordinates.

42. (a) Identify and sketch the curve $xy=9$.

(b) Sketch the graph of the following equations in polar coordinates

(i) $r=3$

(ii) $\theta=\dfrac{\pi}{4}$

(iii) $r=\sin\theta \ (\theta\geq 0)$.

43. (a) Find the parametric equations of the line L passing through the points P (2, 4, –1) and Q (5, 0, 7). Where does the line intersect the xy - plane?

(b) Let L_1 and L_2 given by

$L_1: x=1+4t,\ y=5-4t,\ z=-1+5t$ and

$L_2: x=2+8t,\ y=4-3t,\ z=5+t$ be two lines.

(i) Are the lines parallel?

(ii) Do the lines intersect?

44. (a) Find the equation of the plane through the points $P_1(1, 2, -1)$, $P_2(2, 3, 1)$ and $P_3(3, -1, 2)$.

(b) Determine whether the planes $3x - 4y + 5z = 0$ and $-6x + 8y - 10z - 4 = 0$ are Parallel.

(c) Determine whether the line $x = 3 + 8t$, $y = 4 + 5t, z = -3 - t$ is parallel to the plane $x - 2y + 5z = 12$.

(2 × 15 = 30 Marks)