Reg. No. : ....

Name : ....

Second Semester B.Sc. Degree Examination, September 2022

First Degree Programme under CBCSS

Mathematics

Complementary Course for Statistics

MM 1231.4 : MATHEMATICS II – INTEGRAL CALCULUS AND SEQUENCES AND SERIES

(2021 Admission)

Time : 3 Hours　　　　　　　　　　　　　　　　　　　　Max. Marks : 80

SECTION – A

Answer **all** questions :

1. Evaluate $\int_{-e}^{-1} \frac{1}{x} \, dx$.

2. Find $\int \cos^4 x \, dx$

3. Write the formula for integration by parts.

4. Find the parametric equations for the surface generated by revolving the curve $y = \frac{1}{x}$ about the x-axis.

5. What is a spiral?

6. Write the conversion formulas for spherical coordinate system to rectangular coordinate system.

7. Find the general term of the sequence $\dfrac{1}{2}, -\dfrac{2}{3}, \dfrac{3}{4}, -\dfrac{4}{5}, \ldots$

8. Write the power series for the Bessel function $J_1(x)$.

9. Give an example of a power series in $x - 1$.

10. State the convergence of p - series.

(10 × 1 = 10 Marks)

SECTION – B

Answer **any eight** questions.

11. Evaluate $\int \ln x \, dx$.

12. Evaluate $\displaystyle\int_0^2 x(x^2 + 1)^3 \, dx$.

13. Sketch the region whose area is represented by the definite integral $\displaystyle\int_1^2 2 \, dx$.

14. State the mean-value theorem for integrals.

15. Evaluate : $\displaystyle\int \dfrac{dx}{x^2 + a^2}$ where $a \neq 0$ is a constant.

16. Evaluate : $\displaystyle\int \dfrac{e^{\sqrt{x}}}{\sqrt{x}} \, dx$.

17. Find the area of the region enclosed by $x = y^2$ and $y = x - 2$.

18. Differentiate the Bessel function $J_0(x)$ with respect to x.

P – 1314

19. Find the partial derivatives of $\mathbf{r} = u\mathbf{i} + v\mathbf{j} + (4 - u^2 - v^2)\mathbf{k}$.

20. Define a cardioid.

21. Find the rectangular coordinates of the point whose polar coordinates are $\left(6, \dfrac{2\pi}{3}\right)$.

22. Determine whether the series $\displaystyle\sum_{k=0}^{\infty} \dfrac{5}{4^k}$ converges. If so, find its sum.

23. Find all values of x for which the series $\displaystyle\sum_{k=0}^{\infty} x^k$ converges. Also find the sum of the series for those values of x.

24. State the ratio test for absolute convergence.

25. Find the nth Taylor polynomial for $\dfrac{1}{x}$ about $x = 1$ and express it in sigma notation.

26. Prove that the series $\displaystyle\sum_{k=1}^{\infty} \dfrac{k}{k+1}$ diverges.

(8 × 2 = 16 Marks)

SECTION – C

Answer any **six** questions.

27. Evaluate : $\displaystyle\int \dfrac{x}{x^2 - 4x + 8}\,dx$.

28. Compute the value of the integral : $\displaystyle\int x^2 e^{-x}\,dx$.

29. Find the total area between the curve $y = 1 - x^2$ and the x - axis over the interval $[0, 2]$.

30. Find the volume of the solid generated when the region between the graphs of the equations $f(x) = \dfrac{1}{2} + x^2$ and $g(x) = x$ over the interval $[0, 2]$ is revolved about the x - axis.

31. Let G be the wedge in the first octant that is cut from the cylindrical solid $y^2 + z^2 \le 1$ by the planes $y = x$ and $x = 0$. Compute $\iiint\limits_{G} z\, dV$.

32. Find the area of the region R enclosed between the parabola $y = \dfrac{x^2}{2}$ and the line $y = 2x$.

33. Find the nth Maclaurin polynomial for e^x.

34. Verify whether the sequence : $\left\{ \dfrac{n}{2n+1} \right\}_{n=1}^{\infty}$ converges or diverges.

35. Find the volume of the solid bounded by the cylinder $x^2 + y^2 = 4$ and the planes $y + z = 4$ and $z = 0$.

36. Use the ratio test for absolute convergence to determine whether the series $\sum\limits_{k=1}^{\infty} (-1)^k \dfrac{2^k}{k!}$.

37. Find the sum of the series $\sum\limits_{k=1}^{\infty} \left(\dfrac{3}{4^k} - \dfrac{2}{5^{k-1}} \right)$.

38. Briefly explain the geometric series.

(6 × 4 = 24 Marks)

SECTION – D

Answer any **two** questions.

39. (a) Find the arc length of the curve $y = \dfrac{x^2}{2}$ from $x = 0$ to $x = 1$.

    (b) Find the area of the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$.

40. Evaluate :

    (a) $\displaystyle\int \frac{3x^4 + 3x^3 - 5x^2 + x - 1}{x^2 + x - 2}\, dx$

    (b) $\displaystyle\int_0^{\frac{3}{4}} \frac{dx}{1 - x}$

    (c) $\displaystyle\int_0^{\ln 3} e^x (1 + e^x)^{\frac{1}{2}}\, dx$.

41. Sketch the graph of $r^2 = 4\cos 2\theta$ in polar coordinates.

42. (a) Derive the equation of the tangent plane to the parametric surface $x = uv$, $y = u$, $z = v^2$ at the point where $u = 2$ and $v = -1$.

    (b) Find the volume of the solid within the cylinder $x^2 + y^2 = 9$ and between the planes $z = 1$ and $x + z = 5$.

43. Examine whether the following series converge or divergence.

    (a) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{\sqrt{k} + 1}$

    (b) $\displaystyle\sum_{k=1}^{\infty} \frac{1}{2k^2 + k}$

    (c) $\displaystyle\sum_{k=1}^{\infty} \frac{3k^3 - 2k^2 + 4}{k^7 - k^3 + 2}$.

44. Find the interval of convergence and radius of convergence of the following power series :

(a) $\displaystyle\sum_{k=0}^{\infty} x^k$

(b) $\displaystyle\sum_{k=0}^{\infty} \frac{x^k}{k!}$

(c) $\displaystyle\sum_{k=0}^{\infty} k!\, x^k$

(d) $\displaystyle\sum_{k=0}^{\infty} \frac{(-1)^k x^k}{y^k (k+1)}.$

(2 × 15 = 30 Marks)