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Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, April 2022

First Degree Programme under CBCSS

Statistics

Elective Course

ST 1661.2 : STOCHASTIC PROCESSES

(2014 and 2017 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions. Each carries **1** mark.

1. Find the probability generating function (pgf) of Geometric distribution.
2. What is state space?
3. Define Markov process.
4. Define transition probability matrix.
5. Define auto covariance.
6. What is spectral density?
7. State Chapman-Kolmogorov equation.
8. Define Gaussian process.

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9. What do you mean by a transient state?

10. Give an example of Poisson process.

(10 × 1 = 10 Marks)

SECTION – B

Answer **any eight** questions. Each question carries **2** marks.

11. Define pgf. Explain the method to find mean from it.

12. When a stochastic process is said to have independent increments?

13. Define ergodic state of a Markov chain.

14. Define period of a state in a Markov chain. Give an example of it.

15. Define moving average process.

16. When will you say that a stochastic process $\{x_n; n = 0, 1, 2, \ldots\}$ is a Markov chain.

17. Define a branching process. Give an example of it.

18. Define first order auto regressive process.

19. What is a compound poisson process?

20. What is meant by a trend in time series?

21. State ergodic theorem.

22. Let $\{X_n; n = 0, 1, 2, \ldots\}$ be a branching process and the corresponding offspring distribution has the pgf $P(s) = \frac{2}{3} + \frac{s + s^2}{6}$. Find the probability of extinction of $\{X_n\}$.

(8 × 2 = 16 Marks)

SECTION – C

Answer **any six** questions. Each question carries **4** marks.

23. Define covariance stationary process. Illustrate with an example.

24. What is meant by n-step transition probabilities of a Markov chain? Calculate $P_{12}^{(2)}$ if $P = \begin{bmatrix} 0.5 & 0.3 & 0.2 \\ 0.2 & 0.4 & 0.4 \\ 0.1 & 0.5 & 0.4 \end{bmatrix}$ where the state space is $\{1, 2, 3\}$.

25. Define probability of extinction. Describe the method to find this probability.

26. If $X_1(t)$ and $X_2(t)$ are two independent poisson process with intensity parameters μ_1 and μ_2 respectively, find the conditional distribution of $X_1(t)$ given $X_1(t) + X_2(t)$.

27. Define Poisson process. How it is related to Uniform distribution.

28. Let $\{X_n; n \geq 0\}$ be a Markov chain with states 0, 1 and 2. The transition probability matrix is $\begin{bmatrix} 3/4 & 1/4 & 0 \\ 1/4 & 1/2 & 1/4 \\ 0 & 3/4 & 1/4 \end{bmatrix}$ with initial distribution $P(X_0 = i; i = 0, 1, 2) = 1/3$. Find $P\{X_3 = 1, X_2 = 2, X_1 = 1, X_0 = 2\}$.

29. Define components of time series with examples.

30. Explain the classification of stochastic processes with respect to state space and time with suitable examples.

31. If $P(s)$ is the off spring distribution associated with a branching process $\{X_n\}$ and $P_n(s)$ is that of X_n, Show that $P_n(S) = P(P_{n-1}(s)) = P_{n-1}(P(s))$.

(6 × 4 = 24 Marks)

SECTION – D

Answer **any two** questions. Each question carries **15** marks.

32. Let X_i, $i = 1, 2, ...$ be identically and independently distributed random variables with $P\{X_i = k\} = p_k$ and pgf $P(s)$ and let $S_N = X_1 + X_2 + ... + X_N$ and N is a random variable independent of $X_i's$. Find the distributions of S_N in terms of pgf.

33. (a) Explain branching process. Show that it is stationary.

    (b) The offspring distribution of a branching process is given by $P_0 = \frac{1}{3}, P_1 = \frac{2}{6}, P_2 = \frac{1}{3}$, Find the probability of extinction.

34. Explain the postulates of a Poisson process. Show that for a Poisson process $\{N(t)\}$, $P(N(t) = n)$ is the Poisson pdf of distribution.

35. Let $\{X_n, n \geq 0\}$ be a Markov chain with state space $\{0, 1, 2, 3\}$ having tpm $\begin{bmatrix} 1/3 & 2/3 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 1/2 & 0 & 1/2 & 0 \\ 0 & 0 & 1/2 & 1/2 \end{bmatrix}$. Classify the Markov chain and its states.

(2 × 15 = 30 Marks)
