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N – 1401

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, April 2022

First Degree Programme under CBCSS

Statistics

Elective Course

ST 1661.2 : STOCHASTIC PROCESSES

(2018 and 2019 Admission)

Time : 3 Hours

Max. Marks : 80

SECTION – A

Answer **all** questions. Each question carries **1** mark.

1. Define the state space of a Stochastic Process.
2. When do you say that a Stochastic Process is a continuous time process?
3. Find the probability generating function of a Binomial Random variable.
4. When do you say two states of a Markov Chain are communicative?
5. Define a Markov chain.
6. When is a transition probability matrix (TPM) said to be stochastic?
7. Define a compound Poisson process.
8. Define a Branching process.
9. What is stationarity in Stochastic process?
10. What is irregular variation in a time series data?

(10 × 1 = 10 Marks)

P.T.O.

SECTION – B

Answer **any eight** questions. Each question carries **2** marks.

11. If X and Y are independent Poisson random variables with parameters λ and μ respectively, then what is the distribution of $X+Y$?

12. Establish the expression to get the variance from a probability generating function.

13. If $f(x, y) = Ae^{-(x+y)}, 0 < x, y < \infty$, is the joint probability density function of x and y, find A and also find the marginal pdfs of X and Y and check their independence.

14. Show that recurrence is a class property.

15. When do you say a Markov Chain is irreducible?.

16. Define absorbing Markov Chain with an example.

17. What is the period of a particular state in a Markov Chain?

18. What are the properties of a TPM?

19. For an irreducible Markov Chain, if the stationary distribution exists, then it is unique. Justify.

20. State the ergodic theorem.

21. Distinguish between strict sense and weak sense stationarity.

22. What are the usual Mathematical models used in time series analysis?

23. Define exponential smoothing in a time series data.

24. What is the significance of autocorrelation in time series analysis?.

25. Define the first order autoregressive model.

26. Define the probability of extinction in a branching process.

(8 × 2 = 16 Marks)

SECTION – C

Answer **any six** questions. Each question carries **4** marks.

27. Give an example of a discrete state branching process.

28. Obtain the conditional densities from the joint pdf $f(x, y) = 3 - x - y$, $0 < x, y \leq 1$. Also check the independence of X and Y.

29. Find the Probability generating function (PGF) of a Geometric random variable.

30. For an integer valued r.v X, with $P(X = n) = p_n$ and $P(X \leq n) = q_n$ so that $\sum_{i=0}^{n} p_i = q_n$, then prove that $\sum_{n=0}^{\infty} P(X \leq n)s^n = \frac{G_X(S)}{1-s}, |s| \leq 1$, where $G_X(s)$ is the PGF of X.

31. Explain the various classifications of Stochastic Processes.

32. Distinguish between recurrent and transient states of a Markov Chain.

33. A Markov Chain $\{X(t), t = 0, 1, 2, ...\}$ defined on the state space {1,2,3} has the following TPM. Find the stationary distribution of the chain.

$$P = \begin{pmatrix} 0 & \frac{2}{3} & \frac{1}{3} \\ \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} & 0 \end{pmatrix}.$$

34. Show that for an irreducible Markov Chain, the stationary distribution, if exists, is unique.

35. Discuss on the components of a time series data.

36. Give the names of different methods of measuring trend in time series analysis.

37. Show that for a Gaussian Stochastic process both weak and strong stationarity are equivalent.

38. Establish the Branching process recursion formula based on the probability generating function.

(6 × 4 = 24 Marks)

SECTION – D

Answer **any two** questions. Each question carries **15** marks.

39. Let $X_1, X_2, \ldots$ be a sequence of independent and identically distributed random variables with common PGF as $G_X(S)$. Let N be a random variable independent of the random variables X_i's with PGF as $G_N(s)$ and let $T_N = \sum_{i=1}^{N} X_i$. Then show that the PGF of T_N is $G_{T_N}(s) = G_N(G_x(s))$. Also compute the mean of T_N.

40. A Markov Chain defined with state space S = {1,2,3,4,5} has the following transition probability matrix P. Find (a) all closed classes, (b) irreducible classes, (c) recurrent and (d) transient states.

$$P = \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 4 & 5 \end{array} \\ \begin{pmatrix} 0.3 & 0.4 & 0.4 & 0 & 0 \\ 0 & 0 & 0.3 & 0.4 & 0.4 \\ 0 & 0.3 & 0 & 0.3 & 0.4 \\ 0 & 0 & 0 & 0.4 & 0.6 \\ 0 & 0 & 0 & 0.6 & 0.4 \end{pmatrix} \end{array}$$

41. How do you fit a trend line by the method of least squares in a time series analysis? Also mention the merits and demerits of the method.

42. Define Poisson process. State the important postulates of the Poisson process. If the arrival process is Poisson, then what is the distribution of the inter arrival (waiting) times?

43. Let $\{Z_0 = 1, Z_1, Z_2, \ldots\}$ be a Branching process with family size Y having a Binomial(2, 1/4) distribution. Find the probability that the process will eventually die out.

44. Explain Galton-Watson branching process. Let μ be the expected number of offsprings in each generation in a Galton-Watson branching process. Show that, if $\mu \leq 1$, the process dies out with probability one.

(2 × 15 = 30 Marks)
