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Reg. No. : ....

Name : ....

Fourth Semester B.Sc. Degree Examination, August 2022

First Degree Programme Under CBCSS

Mathematics

Complementary Course of Statistics

MM 1431.4 – MATHEMATICS IV – LINEAR ALGEBRA

(2019 Admission onwards)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions. Each carries **1** mark.

1. Define null space of a matrix A.

2. Write a basis other than standard basis for $\mathbb{R}^2$.

3. Define linear dependence of vectors in $\mathbb{R}^n$

4. What can you say about the eigen values of an $n \times n$ symmetric matrix A if the quadratic form $X^T AX$ is negative definite?

5. Could a 6×9 matrix have a two dimensional null space?

6. True of False : If S spans V and if T is a subset of V that contains more vectors than S, then T is linearly dependent.

P.T.O.

7. True of False : The only 3-dimensional subspace of $\mathbb{R}^3$ is $\mathbb{R}^3$ itself.

8. What is the dimension of the vector space $\mathbb{R}^n$ over $\mathbb{R}$?

9. What is the size of the matrix of a linear transformation from $\mathbb{R}^5$ to $\mathbb{R}^6$ over $\mathbb{R}$?

10. If the null space of a 5×6 matrix A is 4 – dimensional, what is the dimension of the column space of A?

(10 × 1 = 10 Marks)

SECTION – B

Answer **any eight** questions. Each carries **2** marks.

11. Prove that an $n \times n$ matrix with n distinct eigen values is diagonalizable.

12. Let $A = \begin{bmatrix} 1 & 6 \\ 5 & 2 \end{bmatrix}$, $\bar{u} = \begin{bmatrix} 6 \\ -5 \end{bmatrix}$ and $\bar{v} = \begin{bmatrix} 3 \\ -2 \end{bmatrix}$. Are $\bar{u}$ and $\bar{v}$ eigen vectors of A?

13. What are the eigen values of $A = \begin{bmatrix} 3 & 6 & -8 \\ 0 & 0 & 6 \\ 0 & 0 & 2 \end{bmatrix}$?

14. If $\bar{x}$ is an eigen vector for A corresponding to λ, what is $A^3\bar{x}$?

15. Find $T(a_0 + a_1t + a_2t^2)$, if T is the linear transformation from $\mathbb{P}_2$ to $\mathbb{P}_2$ whose matrix relative to $\mathcal{B} = \{1, t, t^2\}$ is $\begin{bmatrix} 3 & 4 & 0 \\ 0 & 5 & -1 \\ 1 & -2 & 7 \end{bmatrix}$.

16. Find the standard matrix A for the dilation transformation $T(\bar{x}) = 3\bar{x}$ for $\bar{x}$ in $\mathbb{R}^2$.

17. Let T be the linear transformation whose standard matrix is $A=\begin{bmatrix}1 & -4 & 8 & 1\\ 0 & 2 & -1 & 3\\ 0 & 0 & 0 & 5\end{bmatrix}$.

Does T map $\mathbb{R}^4$ onto $\mathbb{R}^3$? Is T a one-to-one mapping?

18. Let $S=\{1,t,t^2,\ldots,t^n\}$. Verify that S is a basis for $\mathbb{P}_n$, the set of all polynomials of degree $\leq n$.

19. Determine the dimension of the subspace H of $\mathbb{R}^3$ spanned by the vectors $\vec{v}_1,\vec{v}_2$ and $\vec{v}_3$ where $\vec{v}_1=\begin{bmatrix}2\\-8\\6\end{bmatrix}$, $\vec{v}_2=\begin{bmatrix}3\\-7\\1\end{bmatrix}$, $\vec{v}_3=\begin{bmatrix}-1\\6\\-7\end{bmatrix}$.

20. What is the coordinate matrix of $(1,7,3)$ with respect to the standard basis $\{(1,0,0),(0,1,0),(0,0,1)\}$?

21. Define norm of a vector $\vec{v}$.

22. Show that $\vec{d}$ is orthogonal to $\vec{c}$ where $\vec{d}=\begin{bmatrix}5\\6\\-1\end{bmatrix}$ and $\vec{c}=\begin{bmatrix}4/3\\-1\\2/3\end{bmatrix}$.

23. Give an example of a subset of $\mathbb{R}^3$ that is not a subspace of $\mathbb{R}^3$.

24. If a set $S=\{v_1,v_2,\ldots,v_p\}$ in $\mathbb{R}^n$ contains the zero vector, then prove that the set S is linearly dependent.

25. Compute the quadratic form X^TAX when $A=\begin{bmatrix}5 & 1/3\\ 1/3 & 1\end{bmatrix}$ and $X=\begin{bmatrix}x_1\\x_2\end{bmatrix}$

26. Show that $B=\{(3,0,0),(0,3,0),(0,0,3)\}$ is a basis for $\mathbb{R}^3$.

(8 × 2 = 16 Marks)

SECTION – C

Answer **any six** questions. Each carries **4** marks.

27. Use Cramer's rule to solve the system

$3x_1 - 2x_2 = 6; -5x_1 + 4x_2 = 8$.

28. Find the inverse of the matrix $\begin{bmatrix} 2 & 1 & 3 \\ 1 & -1 & 1 \\ 1 & 4 & -2 \end{bmatrix}$.

29. Find the characteristic polynomial and eigen values of the matrix $\begin{bmatrix} 3 & -2 \\ 1 & -1 \end{bmatrix}$.

30. Let $A = \begin{bmatrix} 7 & 2 \\ -4 & 1 \end{bmatrix}$. Find a formula for A^k, given that $A = PDP^{-1}$.

31. Let $v_1 = \begin{bmatrix} 1 \\ -3 \\ 4 \end{bmatrix}, v_2 = \begin{bmatrix} 6 \\ 2 \\ -1 \end{bmatrix}, v_3 = \begin{bmatrix} 2 \\ -2 \\ 3 \end{bmatrix}, v_4 = \begin{bmatrix} -4 \\ -8 \\ 9 \end{bmatrix}$. Find a basis for the subspace W spanned by $\{v_1, v_2, v_3, v_4\}$.

32. Check whether $\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}$ is a basis for $\mathbb{R}^3$.

33. Find a basis for the null space of the matrix $A = \begin{bmatrix} -3 & 6 & -1 & 1 & -7 \\ 1 & -2 & 2 & 3 & -1 \\ 2 & -4 & 5 & 8 & -4 \end{bmatrix}$

34. Determine whether the set $\left\{ \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} -5 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 7 \\ 0 \\ -5 \end{bmatrix} \right\}$ is a basis for $\mathbb{R}^3$.

35. Is 5 an eigen value of $A = \begin{bmatrix} 6 & -3 & 1 \\ 3 & 0 & 5 \\ 2 & 2 & 6 \end{bmatrix}$?

36. Let T: $\mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation. Then prove that T is one-to-one if and only if $T(x) = 0$ has only the trivial solution.

37. Find the dimensions of the null space and the column space of $A = \begin{bmatrix} 1 & -6 & 9 & 0 & -2 \\ 0 & 1 & 2 & -4 & 5 \\ 0 & 0 & 0 & 5 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$.

38. Let $T(x_1, x_2) = (3x_1 + x_2, 5x_1 + 7x_2, x_1 + 3x_2)$. Show that T is a one-to-one linear transformation Does T maps $\mathbb{R}^2$ onto $\mathbb{R}^3$.

(6 × 4 = 24 Marks)

SECTION – D

Answer **any two** questions. Each carries **15** marks.

39. Find an orthogonal basis for the column space of the matrix $\begin{bmatrix} 3 & -5 & 1 \\ 1 & 1 & 1 \\ -1 & 5 & -2 \\ 3 & -7 & 8 \end{bmatrix}$.

40. Diagonalize the following matrix $A = \begin{bmatrix} 1 & 3 & 3 \\ -3 & -5 & -3 \\ 3 & 3 & 1 \end{bmatrix}$.

41. (a) If a vector space V has a basis $\mathcal{B} = \{b_1, \ldots, b_n\}$, then prove that any set in V containing more than n vectors must be linearly dependent.

(b) Find the dimension of the subspace $H = \left\{ \begin{bmatrix} a - 3b + 6c \\ 5a + 4d \\ b - 2c - d \\ 5d \end{bmatrix} : a, b, c, d \text{ in } \mathbb{R} \right\}$.

42. (a) Let $A = \begin{bmatrix} 4 & -1 & 6 \\ 2 & 1 & 6 \\ 2 & -1 & 8 \end{bmatrix}$. Is 2 an eigen value of A? If so, find a basis for the corresponding eigen space.

(b) Prove that the eigen values of a triangular matrix are the entries on its main diagonal.

43. Show that the mapping T: $\mathbb{P}_2 \rightarrow \mathbb{P}_2$ defined by $T(a_0 + a_1 t + a_2 t^2) = a_1 + 2a_2 t$ is a linear transformation. Also find the $\mathcal{B}$-matrix for T where $\mathcal{B}$ is the basis $\{1, t, t^2\}$.

44. Show that $\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \\ 1 \end{bmatrix} \right\}$ is a basis for a subspace W of $\mathbb{R}^4$ and construct an orthonormal basis for W.

(2 × 15 = 30 Marks)
