(Pages : 4) **K – 2406**

Reg. No. : ....

Name : ....

Third Semester B.Sc. Degree Examination, March 2021

First Degree Programme under CBCSS

Mathematics

Complementary Course for Physics

MM 1331.1 - MATHEMATICS III : CALCULUS AND LINEAR ALGEBRA

(2018 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the **ten** questions are compulsory. They carry **1** mark each.

1. Define Wronskian of 2 functions $y_1(x)$ and $y_2(x)$.

2. Give an example for a Clairaut's equation.

3. State Green's theorem in a plane.

4. Give an example for an Odd function.

5. Define Periodic function.

6. What is the average value of $\sin^2 x$ over a period on $[0, \pi]$?

7. Solve : $y' = -1.5y$.

P.T.O.

8. Define a scalar potential function.

9. Give the integral form for gradient.

10. Define average value of a function.

(10 × 1 = 10 Marks)

SECTION – II

Answer any **eight** questions from among the questions 11 to 22. These questions carry **2** mark each.

11. Find a vector perpendicular to both $\vec{A} = 2\hat{i} + \hat{j} - \hat{k}$ and $\vec{B} = \hat{i} + 3\hat{j} - 2\hat{k}$.

12. Find the complementary function of the equation $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = e^x$.

13. Write the general form of Legendre and Euler linear equations.

14. Solve : $y' = y$.

15. State Green's theorem in a plane.

16. Find the amplitude and frequency of the function $S = 2\sin(4t - 1)$

17. What is the average value of $\sin mx \sin nx$ (over a period)?

18. Write the function $\ln|1 - x|$ as the sum of an even function and an odd function.

19. When is a plane region R said to be simply connected. Give example.

20. Define Linear independence of two vectors. Give examples of two vectors that are linearly independent.

21. Find the characteristic equation corresponding to the matrix $\begin{pmatrix} 1 & 3 \\ 2 & 2 \end{pmatrix}$.

22. Define eigen vector of a linear transformation. Give example.

(8 × 2 = 16 Marks)

SECTION – III

Answer any **six** questions from among the following questions 23 to 31. These questions carry **4** marks each.

23. Solve : $2xy\frac{dy}{dx} - y^2 + x^2 = 0$.

24. Solve : $\frac{dy}{dx} + y\tan x = \cos^3 x$.

25. Show that $(2xy + y - \tan y)\,dx + (x^2 - x\tan^2 y + \sec^2 y + 2)\,dy = 0$ is exact and solve it.

26. Find the volume enclosed between a sphere of radius 'a' and centred at the origin and a circular cone of half angle α with its vertex at the origin.

27. Use Stoke's theorem to show that

$$\int_S d\vec{s} \times \nabla\phi = \oint_C \phi\, d\vec{r}$$

28. Expand $f(x) = \begin{cases} 0, & 0 < x < l \\ 1, & l < x < 2l \end{cases}$ in an exponential Fourier series of period $2l$.

29. Define the Fourier sine Transforms and Fourier cosine transforms.

30. Write an row reduce the augmented matrix for the equations

$$\begin{aligned} x - y + 4z &= 5 \\ 2x - 3y + 8z &= 4 \\ x - 2y + 4z &= 9 \end{aligned}$$

31. Find the cosine of the angle between the planes $x - 2y + 3z = 4$ and $2x + y - z = 5$.

(6 × 4 = 24 Marks)

SECTION – IV

Answer any **two** questions from among the questions 32 to 35. These questions carry **15** marks each.

32. Evaluate the surface integral $I = \int_S \vec{a} \cdot d\vec{s}$ where $\vec{a} = x\hat{i}$ and S is the surface of the hemisphere $x^2 + y^2 + z^2 = a^2$ with $z \geq 0$.

33. Find the Fourier sine series and Fourier cosine series for the function $f(x) = \pi - x$ in $0 < x < \pi$.

34. (a) For $M = \begin{pmatrix} 1 & 0 & -1 \\ -2 & 3 & 0 \\ 1 & -3 & 2 \end{pmatrix}$, find M^{-1}. Use M^{-1} to solve the equations $x - y = 5,\ -2x + 3y = 1,\ x - 3y + 2z = -10$.

    (b) Give examples of 2 matrices A and B such that neither A nor B is the zero matrix, but $AB = 0$.

35. (a) Solve $\dfrac{d^2y}{dx^2} - 3\dfrac{dy}{dx} + 2y = 2e^{-x}$ subject to the boundary conditions $y(0) = 2,\ y'(0) = 1$, using Laplace transform method.

    (b) Solve : $\dfrac{dy}{dx} = \dfrac{y}{x} + \tan\left(\dfrac{y}{x}\right)$.

(2 × 15 = 30 Marks)