(Pages : 3) **L – 6309**

Reg. No. : ....

Name : ....

First Semester M.Sc. Degree Examination, August 2021

Mathematics

MM 213 : ORDINARY DIFFERENTIAL EQUATIONS

(2020 Admission)

Time : 3 Hours Max. Marks : 75

PART – A

Answer **any five** questions from among the questions 1 to 8.

Each question carries **3** marks.

1. Show that $f(x, y) = \sqrt{y}$, does not satisfy a Lipschitz condition on the rectangle $|x| \leq 1$ and $0 \leq y \leq 1$.

2. Find the regular singular points of $x^2 y'' + (2 - x) y' = 0$.

3. What is a hyper geometric series? How is it related to geometric series?

4. Prove that $\Gamma(p+1) = p\Gamma(p)$.

5. Replace the equation $y''' = y'' - x^2 (y')^2$ by equivalent system of first order.

6. Show that $\left\{\begin{array}{l} x = e^{4t} \\ y = e^{4t} \end{array}\right\}$ and $\left\{\begin{array}{l} x = e^{-2t} \\ y = -e^{-2t} \end{array}\right\}$ are solutions of the homogeneous system $\left\{\begin{array}{l} \dfrac{dx}{dt} = x + 3y \\ \dfrac{dy}{dt} = 3x + y \end{array}\right\}$.

P.T.O.

7. Define :

(a) critical points

(b) isolated points

(c) saddle points.

8. What do you mean by linearization of nonlinear systems?

(5×3 = 15 Marks)

PART – B

Answer **all** questions from 9 to 13. Each question carries **12** marks.

9. (a) State and prove Picard's theorem. **12**

OR

(b) (i) Find the power series solution of $y' = 2xy$ and verify your solution by solving the equation directly. **6**

(ii) Solve Legendre's equation. **6**

10. (a) (i) Find the first three terms of the Legendre series of $f(x) = e^x$. **6**

(ii) Prove that $\int_{-1}^{1} P_m(x) P_n(x)\, dx = \left\{ \begin{array}{ll} 0, & if\ m \neq n \\ \dfrac{2}{2n+1}, & if\ m = n \end{array} \right\}$. **6**

OR

(b) (i) Derive Rodrigues' formula. **8**

(ii) Deduce the values of $P_0(x)$, $P_1(x)$, $P_2(x)$ and $P_3(x)$. **4**

11. (a) (i) State and prove the orthogonality property of Bessel's function. **6**

(ii) Express $J_2(x)$, $J_3(x)$ and $J_4(x)$ in terms of $J_0(x)$ and $J_1(x)$. **6**

OR

(b) (i) Prove that $J_{-n}(x) = (-1)^n J_n(x)$. **6**

(ii) Derive Bessel's integral formula. **6**

12. (a) Find the general solution of $\left\{\begin{aligned} \frac{dx}{dt} &= 3x - 4y \\ \frac{dy}{dt} &= x - y \end{aligned}\right\}$. **12**

OR

(b) (i) Show that $\frac{d^2y}{dt^2} > 0$, whenever $\frac{dx}{dt} > 0$. **8**

(ii) What is the meaning of this result in geometrical terms? **4**

13. (a) Find all solutions of the non-autonomous system $\left\{\begin{aligned} \frac{dx}{dt} &= x \\ \frac{dy}{dt} &= x + e^t \end{aligned}\right\}$ and sketch some of the curves defined by the solution. **12**

OR

(b) Find the critical points of $\left\{\begin{aligned} \frac{dx}{dt} &= -y \\ \frac{dy}{dt} &= x. \end{aligned}\right.$ **12**

(5 × 12 = 60 Marks)
