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Reg. No. : ....

Name : ....

First Semester M.Sc. Degree Examination, August 2021

Mathematics

MM 213 – DIFFERENTIAL EQUATIONS

(2005–2019 Admission)

Time : 3 Hours Max. Marks : 75

Instruction : Answer Part A or Part B of each questions.

1. (A) (a) Find the general solution of $y'' + 2y' + 2y = 10\sin 4x$. **5**

(b) Find the third approximation of the solution of the equation $\frac{dy}{dx} = x + y^2$ by Picard's method if $y(0) = 0$. **5**

(c) Using Picard's method, solve the following initial value problem.

$$\frac{dy}{dx} = z\,,\ y(0) = 1\ \frac{dz}{dx} = x^3(y+z),\ z(0) = \frac{1}{2}.$$ **5**

OR

(B) State and prove Picard's theorem. **15**

II. (A) (a) If p is not zero or a positive integer, Show that the series $\sum_{n=1}^{\infty} \frac{p(p-1)\,(p-2).....(p-n+1)}{n!}\, x^n$ converges for $|x| < 1$ and diverges for $|x| > 1$. **5**

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(b) Prove that $\log(1+x) = x\, F(1, 1, 2, -x)$. **5**

(c) Find the general solution of $x(1-x)\, y'' + \left(\frac{3}{2} - 2x\right) y' + 2y = 0$ near the singular point $x = 0$. **5**

OR

(B) (a) Solve Legendre's equation $(1-x^2)\, y'' - 2xy' + p(p+1)\, y = 0$, where p is a constant, in terms of power series in x. **7**

(b) Find two independent Frobenius series solutions of $4xy'' + 2y' + y = 0$. **8**

III. (A) (a) Express $J_4(x)$ in terms of $J_0(x)$ and $J_1(x)$. **5**

(b) Prove that $\int_{-1}^{1} P_m(x)\, P_n(x)\, dx = \begin{cases} 0 \ \text{if} \ m \neq n \\ \frac{2}{2n+1} \ \text{if} \ m = n \end{cases}$ **10**

OR

(B) (a) State and prove Rodrigue's formula. **5**

(b) Find the first three terms of the Legendre series of $f(x) = \begin{cases} 0 \ \text{if} \ -1 \leq x < 0 \\ x \ \text{if} \ 0 \leq x \leq 1 \end{cases}$. **5**

(c) Prove that $\frac{d}{dx}\left[xJ_1(x)\right] = xJ_0(x)$. **5**

IV. (A) (a) Show that $2z = (ax+y)^2 + b$ is a complete integral of $px + qy - q^2 = 0$. **5**

(b) Find the general integral of $(y+1)\, p + (x+1)\, q = z$. **5**

(c) Find the complete integral of $p^2 + q^2 = x + y$. **5**

OR

(B) (a) Show that the Pfaffian differential equation

$(1 + yz)\, dx + x(z - x)\, dy - (1 + xy)\, dz = 0$ is integrable and find its integral. **7**

(b) Find a complete integral of $p^2 x + q^2 y = z$ by Jacobi's method. **8**

V. (A) (a) Reduce the equation $(n - 1)^2 u_{xx} - y^{2n}\, u_{yy} = n y^{2n-1} u_y$, where n is an integer, to a canonical form and solve if possible. **7**

(b) Obtain d' Alembert's solution of the one-dimensional wave equation :

$$\frac{\partial^2 y}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 y}{\partial t^2}, \; -\infty < x < \infty, \; t > 0,$$

$$y(x, 0) = f(x), \; y_t(x, 0) = g(x), \; -\infty < x < \infty.$$ **8**

OR

(B) (a) Establish a necessary condition for the existence of the solution of the Neumann problem. **6**

(b) Suppose that $u(x, y)$ is harmonic in a bounded domain D and continuous in $\overline{D} = D \cup B$. Prove that u attains its maximum on the boundary B of D. **9**

(5 × 15 = 75 Marks)