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Reg. No. : ....

Name : ....

Fourth Semester M.Sc. Degree Examination, March 2021

Mathematics

MM 241 — COMPLEX ANALYSIS – II

(2011 Admission onwards)

Time : 3 Hours Max. Marks : 75

Instructions : All questions carry equal marks.

1. (A) State and prove Arzela – Ascoli theorem. **(15)**

OR

(B) Let G be a region which is not the whole plane and such that every non-vanishing analytic function on G has an analytic square root. If $a \in G$, show that there is an analytic function f on G such that :

(a) $f(a) = 0$ and $f'(a) > 0$;

(b) f is one-one;

(c) $f(G) = D = \{z : |z| < 1\}$. **(15)**

(A) Let G be a region and let $\{a_j\}$ be a sequence of distinct points in G with no limit point in G and let $\{m_j\}$ be a sequence of integers. Show that there is an analytic function f defined on G whose only zeros are at the points a_j; further more, a_j is a zero of f of multiplicity m_j. **(15)**

OR

P.T.O.

(B) (a) Derive the Gauss's formula. **(8)**

(b) Show that $\left\{\left(1+\frac{z}{n}\right)^n\right\}$ converges to e^z in H($\mathbb{C}$). **(7)**

3. (A) (a) Show that, for $\operatorname{Re} z > 1$, $\xi(z)\Gamma(z) = \int_0^\infty (e^t - 1)\, t^{z-1}\, dt$. **(8)**

(b) If $\operatorname{Re} z > 1$, then prove that $\xi(z) = \prod_{n=1}^{\infty}\left(\frac{1}{1-p_n^{-z}}\right)$, where $\{p_n\}$ is the sequence of prime numbers. **(7)**

OR

(B) State and prove Runge's theorem. **(15)**

4. (A) State and prove schwarz reflection principal. **(15)**

OR

(B) State and prove Monodromy theorem. **(15)**

5. (A) (a) State and prove the second version of the Maximum principle. **(8)**

(b) If G is a region and if $\{u_n\}$ is a sequence in Har(G) such that $u_1 \le u_2 \le .....$, then show that either $u_n(z) \to \infty$ uniformly on compact subsets of G or $\{u_n\}$ converges in Har(G) to a harmonic function. **(7)**

OR

(B) If is an entire function of genus μ, then show that for each positive number α there is a number r_0 such that for $|z| > r_0$ $|f(z)| < \exp\left(\alpha |z|^{\mu+1}\right)$. **(15)**

(5 × 15 = 75 Marks)
