(Pages : 3) **L – 5416**

Reg. No. : ....

Name : ....

Fourth Semester M.Sc. Degree Examination, March 2021

Mathematics

MM 242 : FUNCTIONAL ANALYSIS – II

(2017 Admission onwards)

Time : 3 Hours Max. Marks : 75

Instruction : Answer either Part A or Part B of each question.

All questions carry equal marks.

1. (A) (a) Let X be a normed space and $A \in CL(X)$. Prove that if X is infinite dimensional, then $0 \in \sigma_a(A)$. **8**

(b) Let X be a normed space and $A \in CL(X)$. Prove that the eigen spectrum and the spectrum of A are countable sets and have 0 as the only possible limit point. **7**

OR

(B) (a) Let X be a normed space and $A : X \to X$. Let $0 \neq k \in K$ and Y be a proper closed subspace of X such that $(A - kI)(X) \subset Y$. Prove that there is some $x \in X$ such that $\|x\| = 1$ and for all $y \in Y$, $\|A(x) \cdot A(y)\| \geq \frac{|k|}{2}$.

(b) Let X be a normed space and $A \in CL(X)$. Prove that $\sigma(A') = \sigma(A)$.

P.T.O.

2. (A) (a) Show that $C^1([a, b])$ is dense in H, where $H = \{x \in C([a, b]) : x$ is absolutely continuous on $[a, b]$ and $x' \in C^2([a, b])^2\}$. **4**

(b) State and prove Parallelogram Law. **3**

(c) State and prove Bessel's inequality. **8**

OR

(B) (a) State and prove Gram-Schmidt orthogonalization. **8**

(b) Let X be an inner product space. If E is an orthogonal subset of X and $0 \notin E$, Prove that E is linearly independent. **4**

(c) Derive Parseval formula. **3**

3. (A) (a) Let X be an Inner Product Space. Let F be an subspace of X and $x \in X$. Prove that $y \in F$ is a best approximation from F to x if and only if $(x - y) \perp F$. **8**

(b) Let E be a nonempty closed convex subset of a Hilbert space H. Prove that for each $x \in H$, there exists a unique best approximation from E to x. **7**

OR

(B) (a) Let X be an Inner Product Space and $f \in X'$. If $\{u_\alpha\}$ is an orthonormal set in X, prove that $\sum_n |f(u_n)|^2 \leq \|f\|^2$. **8**

(b) Let (x_n) be a sequence in a Hilbert space H. If (x_n) is bounded, then it has a weak convergent subsequence. **7**

4. (A) (a) Let H be a Hilbert space and $A \in BL(H)$. Prove that there is a unique $B \in BL(H)$ such that for all $x, y \in H$, $\langle A(x), y \rangle = \langle x, B(y) \rangle$. **8**

(b) Let H be a Hilbert space and $A \in BL(H)$. If $R(A) = H$, prove that A^* is bounded below and $R(A^*) = H$. **7**

OR

(B) (a) Let H be a Hilbert space and $A \in BL(H)$. If A is self-adjoint, prove that $\|A\| = \sup\{|\langle A(x), x \rangle| : x \in H, \|x\| \leq 1\}$.

(b) State and prove Generalized Schwarz inequality.

5. (A) (a) Let $H \neq \{0\}$ and $A \in BL(H)$ be self-adjoint. Prove that $\{m_A, M_A\} \subset \sigma_a(A) = \sigma(A) \subset [m_A \cdot M_A]$.

(b) State and prove Ritz method.

OR

(B) (a) Let $A \in BL(H)$. If each A_n is compact operator on H and $\|A_n - A\| \to 0$, prove that A is compact.

(b) Let A be a compact operator and $H \neq \{0\}$. If A is self-adjoint, prove that $\|A\|$ or $-\|A\|$ is an eigenvalue of A.

(5 × 15 = 75 Marks)