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Reg. No. : ....

Name : ....

Second Semester B.Sc. Degree Examination, September 2022

First Degree Programme under CBCSS

Statistics

Complementary Course for Mathematics

ST 1231.1 : PROBABILITY AND RANDOM VARIABLES

(2020 Admission Onwards)

Time : 3 Hours Max. Marks : 80

PART – A

Answer **all** questions. Each question carries 1 mark.

1. Define mutually exclusive events.
2. Describe probability space.
3. Define conditional probability.
4. Give an example of a continuous sample space.
5. When will you say that two events are independent?
6. Describe descrete random variables.
7. Examine whether the following is a probability mass function or not

   $f(x) = \frac{1}{2^x}, x = 1, 2.....$

P.T.O.

8. A random variable X take values 0 and 1 with probabilities $\frac{1}{2}$. Find $E(X)$.

9. Give any two properties of expectation.

10. Define moment generating function.

(10 × 1 = 10 Marks)

PART – B

Answer **any eight** questions. Each question carries **2** marks.

11. Discuss classical approach to probability. Give any one of its draw backs.

12. Describe multiplication theorem on probability.

13. Distinguish between equally likely and exhaustive events.

14. A bag contains 5 red and 4 black balls. Two balls are drawn at random. What is the probability that both of them are red?

15. Discuss any two properties of moment generating function.

16. If A and B are any two events such that $P(A)=\frac{1}{2}$; $P(B)=\frac{1}{3}$ and $P(A\cap B)=\frac{1}{4}$. Find (a) $P(A\cup B)$; (b) $P(\overline{A}\cup\overline{B})$.

17. A random variable X has the following probability function. Find the value of K.

X:	0	1	2	3	4	5	6	7
$P(X=x)$:	0	k	$2k$	$2k$	$3k$	k^2	$2k^2$	$7k^2+k$

18. In six tosses of an unbiased coin, let X denote the number of heads occurred. Find $E(X)$.

19. Find the characteristics function of X with probability function

$$P(X=x)=\binom{n}{x}P^x(1-p)^{n-x},\ x=0,1,.....n.$$

20. Discuss conditional expectation.

21. The joint probability density function of (X, Y) is $f(x, y) = 8xy, 0 < x < y < 1$. Find the marginal distribution of X.

22. With usual notations show that

$$E(E(X/Y)) = E(X)$$

23. Define independence of random variables.

24. For a random experiment throwing a die let $A = \{1, 2, 3\}$ and $B = \{3, 4, 5, 6\}$. Find

(a) $P(\overline{A})$

(b) $P(A \cup \overline{B})$

(c) $P(\overline{A} \cup B)$

25. Given $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{4}$ and $P(A/B) = \frac{1}{6}$ find (a) $P(B/A)$ (b) $P(A \cap B)$.

26. Find the characteristics function of a random variable X with probability function $f(x, \theta) = \theta\, e^{-\theta x}, x > 0$.

(8 × 2 = 16 Marks)

PART – C

Answer **any six** questions. Each question carries **4** marks.

27. Distinguish between pairwise independence and mutual independence.

28. Show that conditional probability satisfies the axioms of probability.

29. Three urns contains

1 White 2 black and 3 red

2 White 1 black and 1 red

4 White 2 black and 3 red

balls. One urn is chosen at random and two balls are selected. They happens to be white. What is the probability that they came from urn II?

30. Two dice are thrown at random. What is the probability that the sum on the phases is

(a) greater than 8

(b) neither 6 or 7.

31. Describe distribution function. State and establish two of its properties.

32. X and Y are distributed according to

$f(x, y) = e^{-(x+y)}, \ x > 0, y > 0.$

Examine whether X and Y are independent or not.

33. Establish any two properties of characteristics function.

34. Describe raw and central moments. Establish the relation between them.

35. Find the moment generating function of the random variable X with probability function.

$f(x) = \frac{e^{-\lambda}\lambda^x}{x!}, x = 0, 1, 2......$

Hence find the first two raw moments of X.

36. Give the empirical definition of probability. Also explain its draw-backs.

37. With usual notations show that for any three events A, B and C.

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$

38. State and prove Baye's theorem.

(6 × 4 = 24 Marks)

PART – D

Answer **any two** questions. Each question carries **15** marks.

39. (a) Given $P(A) = P_1, P(B) = P_2$ and $P(A \cap B) = P_3$. Express the following in terms of P_1, P_2 and P_3

(i) $P(\overline{A} \cup \overline{B})$

(ii) $P(\overline{A} \cap B)$;

(iii) $P(\overline{A} \cup B)$

(iv) $P(A \cup B)$;

(v) $P(\overline{A} \cap \overline{B})$.

(b) Let A and B be two events such that $P(A) = \frac{3}{4}$ and $P(B) = \frac{5}{8}$. Show that

(i) $P(A \cup B) \geq \frac{3}{4}$

(ii) $\frac{3}{8} \leq P(A \cap B) \leq \frac{5}{8}$

40. (a) A and B alternatively throw a pair of dice. A wins if he throws 6 before B throws 7 and B wins if he throws 7 before A throws 6. Find the probability of A winning the game.

(b) Two dice are thrown and three events are defined as

A : Odd face with first dice

B : Odd face with second dice

C : Sum on the two faces is odd

Examine whether A, B and C are mutually independent or not.

41. (a) In a bolt factory machine A, B and C manufactured respectively 25%, 35% and 40% of the total. Of their total output 5, 4, 2 percents respectively are defective bolts. A bolt is drawn at random and is found to be defective. What are the probabilities that it was manufactured by machines A, B and C.

(b) With usual notations show that $P(A \cap B \cap C) = P(A) \cdot P(B/A) \cdot P(C/A \cap B)$

42. (a) A random variable X has the probability function $f(x) = kx(2-x),\ \ 0 < x \leq 2$. Find the value of k. Also find

(i) $P\left(X \leq \frac{1}{2}\right)$

(ii) $P\left\{\frac{1}{2} \leq X \leq \frac{3}{2}\right\}$

(iii) $P(X \geq 1)$

(b) Explain the method of transformation of one dimensional random variable. A random variable X has the probability function $f(x) = e^{-x}$, $x > 0$. Find the probability density function of

(i) $y = x^2$

(ii) $y = 3x + 5$

43. (a) X and Y are two random variables with probability function $f(x, y) = \frac{1}{27}(x + 2y),\ x = 0, 1, 2;\ y = 0, 1, 2$. Find the marginal distributions of X and Y.

(b) The joint probability density function of X and Y is given by.

$(f(x, y) = 2, 0 < x < 1, 0 < y < x)$

(i) Find the marginal distribution of X and Y.

(ii) Find the conditional distribution of X given $Y = y$. Also examine the independence of x and Y.

44. (a) State and prove Cauchy-Schwartz inequality.

(b) Given $f(x, y) = 21x^2y^3, 0 < x < y < 1$
$= \text{otherwise}$

Find the conditional mean and conditional variance of X given $Y = y$.

(2 × 15 = 30 Marks)