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Reg. No. : ....

Name : ....

Second Semester B.Sc. Degree Examination, September 2022

First Degree Programme under CBCSS

Statistics

Complementary Course for Mathematics

ST 1231.1 : PROBABILITY AND RANDOM VARIABLES

(2018 & 2019 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions, each carries **1** mark.

1. What do you mean by sample space?

2. State Bayes Theorem.

3. Give an example of a discrete random variable.

4. Let X be a random variable with probability mass function given below. Find k.

X	0	1	2
$P(X = x)$	k	$2k$	$3k$

5. Discuss the condition for independence of X and Y in terms of marginals.

6. If a random variable X has mean 3 and standard deviation 4. What will be the variance of $Y = 2X + 5$.

P.T.O.

7. Give an example of a random variable whose expectation does not exist.

8. What is the relationship between distribution function and density function?

9. State the multiplication theorem in probability.

10. A card is drawn from a well shuffled pack of 52 cards. What is the probability of the card being black?

(10 × 1 = 10 Marks)

SECTION – B

Answer **any eight** questions each carries **2** marks.

11. Discuss on the counting rules combination and permutation.

12. Let $P(A) = 0.5$ and $P(B) = x$ and $P(A \text{ or } B) = 0.8$. Find the value of x for which A and B are independent.

13. Define mutually exclusive events. Give an example.

14. Two unbiased dice are tossed. What is the probability that the sum of points scored on the two dice is 8?

15. Discuss the properties of distribution function.

16. A problem in Statistics is given to three students A, B and C whose chances of solving it are $\frac{1}{2}, \frac{1}{3}$ and $\frac{1}{4}$ respectively. What is the probability that the problem will be solved?

17. Prove that an event is independent of itself if and only if $P(A) = 0$.

18. Let X be a random variable with probability mass function given below. Find the distribution function of X

X	0	1	2	3	4	5
$P(X = x)$	1/16	1/16	10/16	1/16	2/16	1/16

19. If $f(x) = 1/n$ for $x = 1, 2, \ldots. n$. Find its characteristic function.

20. Define bivariate moments.

21. Define moment generating function of a distribution. Show how it generates moments.

22. Let X be a random variable with p.m.f. given below, find the expected value of $Z = (X-1)^2$.

X	0	1	2	3
$P(X=x)$	1/3	1/2	1/24	1/8

(8 × 2 = 16 Marks)

SECTION – C

Answer **any six** questions each carries **4** marks.

23. Discuss the importance of probability in Science and Industry.

24. Explain the axiomatic approach in probability.

25. State and prove addition theorem in probability for two events.

26. If $P(A) = 0.30$, $P(B) = 0.78$ and $P(A \cap B) = 0.16$. Find $P(A^c \cap B^c), P(A^c \cup B^c), P(A \cap B^c)$?

27. Prove that mutual independence of events implies pairwise independence of events. Also show that the converse is not true.

28. From a box containing 25 items 5 of which are defectives. 4 are chosen at random. Find the probability distribution of the number of defectives obtained.

29. State and prove Cauchy Schwartz inequality.

30. State and prove addition theorem on expectation for two variables X and Y.

31. A continuous random variable Y has the pdf $f(x) = kx^2$, $0 < x < 1$. Find a and b such that $P(X \le a) = P(X > a)$ and $P(X > b) = 0.05$.

(6 × 4 = 24 Marks)

SECTION – D

Answer **any two** questions, each carries **15** marks.

32. (a) Discuss the procedure and formula to find $E(X)$ and $E(Y)$ by using marginal and joint density function.

(b) Let (X, Y) be a pair of random variables with joint pdf given below, find k and check the independence of X and Y

$$f(x, y)=\begin{cases} k(1+x+y)^{-n}, & x>0, y>0, n>2 \\ 0, & \text{elsewhere} \end{cases}$$

33. Find the correlation between X and Y if the joint probability distribution of X and Y is $f(x,y)=\begin{cases} 2-x-y, & 0<x<1 \text{ and } 0<y<1, \\ 0, & \text{otherwise} \end{cases}$

34. If X is a random variable having the p.d.f $f(x)=q^{x-1}p,\ x=1,2,3......$ such that $p+q=1$. Find the m.g.f. and hence find its mean and variance.

35. (a) Discuss union, intersection and complementation of events with suitable example.

(b) Suppose we have a sample space with five equally likely experimental outcomes $E_1, E_2 ..., E_5$. Let $A=\{E_1, E_2\}$, $B=\{E_3, E_4\}$ and $C=\{E_2, E_3, E_5\}$, find

(i) $P(A), P(B), P(C)$

(ii) $A \cup B$, Are A and B mutually exclusive?

(iii) $A^c, C^c, P(A^c)$ and $P(C^c)$

(iv) $A \cup B^c$ and $(P(A \cup B^c)$

(v) $P(B \cup C)$.

(2 × 15 = 30 Marks)
