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Reg. No. : ....

Name : ....

First Semester B.Sc. Degree Examination, November 2019

First Degree Programme under CBCSS

Mathematics

Core course

MM 1141 : METHODS OF MATHEMATICS

(2018 Admission Onwards)

Time : 3 Hours Max. Marks : 80

SECTION – I

(All the questions are compulsory. Each question carries **1** mark.)

1. Define local linear approximation of f at x_0.

2. Define percentage error in measurement.

3. Give the differential formula for quotient rule of differentiation.

4. Define inflection points of a curve.

5. How can you interpret the sign of acceleration?

6. Give the formula for average value of a function.

7. What is the volume of a solid bounded by $x = a$ and $x = b$ having a cross sectional area, $A(x)$

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8. Give the formula for work done by a variable force F in moving and object over $[a, b]$

9. Define Hyperbolic sine and cosine functions

10. Evaluate $\int_0^{\infty} \frac{dx}{x^3}$

(10 × 1 = 10 Marks)

SECTION – II

(Answer **any eight** questions. Each question carries **2** marks)

11. Suppose that x and y are differentiable functions of t and are related by the equation $y = x^3$ find $\frac{dy}{dt}$ at time $t = 1$ if $x = 2$ and $\frac{dx}{dt} = 4$ at time $t = 1$.

12. Stat the L'Hospital's Rule for form 0/0.

13. Find the intervals on which $f(x) = x^3$ is increasing and the intervals on which it is decreasing.

14. Evaluate $\lim_{x\to\infty} \frac{x^{\frac{-4}{3}}}{\sin\left(\frac{1}{x}\right)}$.

15. State a sufficient condition for $f(x)$ to be concave up or concave down.

16. State Mean value theorem for $f(x)$.

17. Find the average value of the function $f(x) = \sqrt{x}$ over the interval $[1,4]$.

18. State the formula for Volume by Washer method.

19. A triangular lamina with vertices (0,0), (0,1) and (1,0) has density $\delta = 3$. Find its total mass.

20. The face of a dam is a vertical rectangle of height 100ft and width 200 ft. Find the total fluid force exerted on the face when the water surface is level with the top of the dam.

21. Prove that $\cosh^2 x - \sinh^2 x = 1$.

22. Evaluate $\int_0^\infty \frac{dx}{1+x^2}$.

(8 × 2 = 16 Marks)

SECTION – III

(Answer **any six** questions. Each question carries 4 marks)

23. Suppose that the side of a square is measured with a ruler to be 10 inches with a measurement error of at most $\pm\frac{1}{32}$ in. Estimate the error in the computed area of the square.

24. Define critical points and find all critical points of $f(x) = 3x^{\frac{5}{3}} - 15x^{\frac{2}{3}}$.

25. Find the relative extrema of $f(x) = 3x^5 - 5x^3$.

26. Find the absolute maximum and minimum values of the function $f(x) = 2x^3 - 15x^2 + 36x$ on the interval [1,5], and determine where these values occur.

27. An open box is to be made from a 16-inch by 30-inch piece of cardboard by cutting out squares of equal size from the four corners and bending up the sides. What size should the squares be to obtain a box with the largest volume?

28. Suppose that a particle moves along a coordinate line so that its velocity at time t is $v(t) = 2 + \cos t$. Find the average velocity of the particle during the time interval $0 \le t \le \pi$.

29. Find the arc length of the curve $y = x^{\frac{3}{2}}$ from (1,1) to $(2,2\sqrt{2})$ by integrating with respect to x.

30. Use cylindrical shells to find the volume of the solid generated when the region R under $y = x^2$ over the interval [0,2] is revolved about the line $y = -1$.

31. Evaluate $\int_0^\infty (1-x)e^{-x}\,dx$.

(6 × 4 = 24 Marks)

SECTION – IV

(Answer **any two** questions. Each question carries **15** marks)

32. (a) Suppose that liquid is to be cleared of sediment by allowing it to drain through a conical filter that is 16cm high and has a radius of 4cm at the top. Suppose also that the liquid is forced out of the cone at a constant rate of $2cm^3/\text{min}$. Find a formula that expresses the rate at which the depth of the liquid is changing in terms of the depth.

(b) Find a point on the curve $y = x^2$ that is closest to the point (18,0).

33. (a) Evaluate $\lim_{x \to 0}(1+\sin x)^{\frac{1}{x}}$. using L'Hospital's Rule.

(b) Find the radius and height of the right circular cylinder of largest volume that can be inscribed in a right circular cone with radius 6 inches and height 10 inches.

34. (a) A coin is released from rest near the top of a building at a point that is 1250 ft above the ground, Assuming that the free-fall model applies and $g = 32ft/s^2$ how long does it take for the coin to hit the ground, and what is its speed at the time of impact?

(b) Derive the formula for the volume of a right pyramid whose altitude is *h* and whose base is a square with sides of length *a*.

35. (a) Find the centroid of the region *R* enclosed between the curves $y = x^2$ and $y = x+6$.

(b) Prove that $\sin^{-1}{}_x = \ln\left[x + \sqrt{x^2+1}\right]$ and $\tanh^{-1} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$.

(2 × 15 = 30 Marks)
