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Reg. No. : ....

Name : ....

Third Semester B.Sc. Degree Examination, March 2022

First Degree Programme under CBCSS

Mathematics

Complementary Course for Physics

MM 1331.1 : MATHEMATICS III – CALCULUS AND LINEAR ALGEBRA

(2019 and 2020 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the **ten** questions are compulsory. They carry **1** mark each.

1. Find the degree of the ODE $\frac{d^2y}{dx^2}+x\left(\frac{dy}{dx}\right)^{3/2}+x^2y=0$

2. What is the auxillary equation of $\frac{d^2y}{dx^2}-2\frac{dy}{dx}+y=e^x$.

3. Write the general form of Euler's linear equation.

4. Prove that $\vec{a}=(xy^2+z)\vec{i}+(x^2y+2)\vec{j}+x\vec{k}$ is conservative.

5. State Green's theorem.

6. Find the average value of $\sin^2 nx$ on $(-\pi,\pi)$.

P.T.O.

7. What are the Fourier coefficients of an even function $f(x)$ in the interval $(-l, l)$.

8. Evaluate $\begin{vmatrix} 1 & -5 & 2 \\ 7 & 3 & 4 \\ 2 & 1 & 5 \end{vmatrix}$.

9. Find the rank of the matrix $\begin{pmatrix} 1 & 1 & 2 \\ 2 & 4 & 6 \\ 3 & 2 & 5 \end{pmatrix}$.

10. Find the trace of the matrix $\begin{pmatrix} 1 & 0 & -1 \\ -2 & 3 & 0 \\ 1 & -3 & 2 \end{pmatrix}$

SECTION – II

Answer **any eight** questions from among the questions **11** to **26**. These questions carry **2** marks each.

11. Solve $\dfrac{dy}{dx} = x + xy$.

12. Solve $x\dfrac{dy}{dx} + 3x + y = 0$.

13. Solve $y = px + p^2$.

14. Find the complementary function of the equation $\dfrac{d^2y}{dx^2} + 2\dfrac{dy}{dx} + y = 4e^{-x}$.

15. Find a solution of $(x^2 + x)\dfrac{dy}{dx}\dfrac{d^2y}{dx^2} - x^2y\dfrac{dy}{dx} - x\left(\dfrac{dy}{dx}\right)^2 = 0$.

16. Evaluate the line integral $I = \int a \cdot dr$ where $\vec{a} = (x+y)\vec{i} + (y-x)\vec{j}$ along the parabola $y^2 = x$ from (1, 1) to (4, 2) in the xy plane.

17. Find the vector area of the surface of the hemisphere $x^2+y^2+z^2=a^2$, $z \geq 0$, by evaluating the line integral $S=\frac{1}{2}\oint_C \bar{r} \times d\bar{r}$ around its perimeter.

18. Find an expression for the angular momentum of a solid body rotating with angular velocity w about an axis through the origin.

19. Define even and odd functions and give examples.

20. What are the Dirichlet conditions for the existence of the Fourier series of a periodic function?

21. Define Fourier transforms.

22. Whether given set of equations has exactly one solution, no solution or an infinite, set of solutions $x-2y+13=0$, $y-2x=17$.

23. Find the direction of the line of intersection of the plane $x-2y+3z=4$ and $2x+y-z=5$.

24. Show that the functions 1, $x, \sin x$ are linearly independent.

25. Find the inverse of the matrix $\begin{pmatrix} 2 & 1 \\ 0 & -3 \end{pmatrix}$.

26. Define symmetric matrix and prove that AA^T is a symmetric matrix for any matrix A.

SECTION – III

Answer **any six** questions from among the following questions **27** to **38**. These questions carry **4** marks each.

27. Solve : $\frac{dy}{dx}=-\frac{2}{y}-\frac{3y}{2x}$.

28. Solve : $\frac{dy}{dx}=(x+y+1)^2$.

29. Solve $x^2\dfrac{d^2y}{dx^2}+x\dfrac{dy}{dx}-4y=0$.

30. Solve $(1-x^2)\dfrac{d^2y}{dx^2}-3x\dfrac{dy}{dx}-y=1$.

31. The vector field $\vec{F}$ is given by $\vec{F}=2xz\vec{i}+2yz^2\vec{j}+(x^2+2y^2z-1)\vec{k}$. Calculate $\nabla\times\vec{F}$ and deduce that $\vec{F}$ can be written $\vec{F}=\nabla\phi$. Determine the form of ϕ.

32. Show that the area of a region R enclosed by a closed curve C is given by $A=\dfrac{1}{2}\oint x\,dy-y\,dx=\oint_C x\,dy=-\oint_C y\,dx$.

33. Find the Fourier series of the function

$$f(x)=\begin{cases}0 & -\pi<x<0\\ 1 & 0<x<\pi\end{cases}$$

34. Expand $f(x)=\begin{cases}-1 & -l<x<0\\ 1 & 0<x<l\end{cases}$ as a Fourier series.

35. Find the equation of the plane through the three points $A(-1,1,1)$, $B(2,3,1)$, $C(0,1,-2)$.

36. Use Crammer's rule to solve $2x-z=2$; $6x+5y+3z=7$; $2x-y=4$.

37. Find the eigen values and any one eigen vector of $\begin{pmatrix}1 & 2 & 2\\ 2 & 3 & 0\\ 2 & 0 & 3\end{pmatrix}$.

38. Solve the set of homogeneous equations by row reducing the matrix $2x+3z=0$, $4x+2y+5z=0$, $x-y+2z=0$.

SECTION – IV

Answer **any two** questions from among the questions **39** to **44**. These questions carry **15** marks each.

39. Use Green's function to solve $\frac{d^2y}{dx^2}+y=\operatorname{cosec} x$ subject to the boundary conditions $y(0)=y\left(\frac{\pi}{2}\right)=0$.

40. Express the equation $\frac{d^2y}{dx^2}+4x\frac{dy}{dx}+(4x^2+6)y=e^{-x^2}\sin 2x$ in canonical form and hence find its general solution.

41. Given the vector field $\vec{a}=y\vec{i}-x\vec{j}+z\vec{k}$, verify Stoke's theorem for the hemispherical surface $x^2+y^2+z^2=a^2,\ z\geq 0$

42. Let $f(x)=\begin{cases}1 & 0<x<\frac{1}{2}\\ 0 & \frac{1}{2}<x<1\end{cases}$

    Find :

    (a) a Fourier sine series

    (b) a Fourier cosine series

    (c) a Fourier series (exponential series, whose period) is 1

43. (a) Find the distance from the point $P(1,-2,3)$ to the plane $3x-2y+z+1=0$.

    (b) Find the distance between the lines $\vec{r}=\vec{i}-2\vec{j}+(i-k)t$ and $\vec{r}=2\vec{j}-\vec{k}+(\vec{j}-\vec{i})t$.

    (c) Prove that $\begin{vmatrix}1 & a & bc\\ 1 & b & ac\\ 1 & c & ab\end{vmatrix}=\begin{vmatrix}1 & a & a^2\\ 1 & b & b^2\\ 1 & c & c^2\end{vmatrix}=(c-a)(b-a)(c-b)$.

44. (a) Solve $x - z = 5$, $-2x + 3y = 1$, $x - 3y + 2z = -10$ by the method of finding the inverse of the coefficient matrix.

(b) Find out whether the given vectors are dependent or independent; if they are dependent find a linearly independent subset.

$(1, -2, 3), (1, 1, 1), (-2, 1, -4)\ (3, 0, 5)$