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Reg. No. : ....

Name : ....

Fourth Semester M.Sc. Degree Examination, September 2019

Mathematics

MM 241 : COMPLEX ANALYSIS II

(2011 Admission onwards)

Time : 3 Hours Max. Marks : 75

All questions carry equal marks :

I. (A) (a) If G is open in $\mathbb{C}$ then prove that there is a sequence $\{K_n\}$ of compact subjects of G such that $G = \bigcup_{n=1}^{\infty} K_n$. Moreover, the sets K_n be chosen to satisfy the following conditions:

(i) K_n C int K_{n+1};

(ii) $K \subset G$ and K compact implies $K \subset K_n$ for some n;

(iii) Every component of $C_\infty - K_n$ contains a component of $\mathbb{C}_\infty$. – G.

8

(b) Prove that R a family F is $H(G)$ is normal iff F is locally bounded. 7

OR

(B) State and prove Arzela - Ascoli theorem. 15

P.T.O.

II. (A) (a) Let (X,d) be a compact metric space and let $\{g_n\}$ be a sequence of continuous functions from X into C such that $\sum g_n(x)$ converges absolutely and uniformly for x in X. Then prove that the product $\prod_{n=1}^{\infty}(1+g_n(x))$ converges absolutely and uniformly for x in X. Also prove that there is an integer N_0. Such that $f(x)=0$ iff $g_n(x)=-1$ for some $n, 1 \leq n \leq N_0$. **8**

(b) Let f be a function defined on $(0,\infty)$ such that $f(x)>0$ for all $x>0$. Suppose that f has the following properties:

(i) $\log f(x)$ is convex function;

(ii) $f(x+1)=xf(x)$ for all x;

(iii) $f(1)=1$. Then prove that $f(x)=\Gamma(x)$ for all x. **7**

OR

(B) (a) If $|z|\leq 1$ and $p\geq 0$ then prove that $|1-E_p(z)|\leq |z|^{p+1}$. **8**

(b) (i) Prove that $\{(1+z/n)^n\}$ converges to e^z in $H(c)$

(ii) If $t\geq 0$ then prove that $(1-t/n)^n \leq e^{-t}$ for all $n \geq t$. **7**

III. (A) (a) For $R_e\, z>1$, prove that $\varsigma_{(z)}\sqrt{(z)}=\int_0^{\infty}(e^t-1)^{-1}t^{z-1}\,dt$. **8**

(b) Let K be a compact subset of the region G; then prove there are straight line segments $r_1, r_2, \cdots, r_m$ in G - K such that for every function f in $H(G)$, $f(z)=\sum_{k=1}^{n}\frac{1}{2\pi i}\int_{r_k}\frac{f(w)}{w-z}dw$ for all Z in K. **7**

OR

(B) (a) State and prove Euler's theorem. **8**

(b) Let γ be a rectifiable curve and let K be a compact set such that $K \cap \{\gamma\} = \square$ If f is continuous function on $\{\gamma\}$ and $E > 0$ then prove there is a rational function $R(z)$ having all its poles on $\{\gamma\}$ and such that $\left| \int_\gamma \frac{f(w)}{w-z}\, dw - R(z) \right| < E$ for all z in k. **7**

IV. (A) (a) State and prove schwarz reflection principle **8**

(b) Let $\gamma : [0, 1] \to C$ be a path and let $\{(f_t, D_t) : 0 \le t \le 1\}$ be an analytic continuation along γ. For $0 \le t \le 1$ let $R(t)$ be the radius of convergence of the power series expansion of f_t about $z = \gamma(t)$. Then prove that either $R(t) \equiv \infty$ or $R : [0,1] \to (0, \infty)$ is continuous. **7**

OR

(B) State and prove Monodromy theorem. **15**

V. (A) Let $D = \{z / |z| < 1\}$ and suppose that $f : \partial D \to R$ is a continuous function. Then prove that there is a continuous function $u : \overline{D} \to R$ such that

(a) $u(z) = f(z)$ for Z in ∂D;

(b) u is harmonic in D.

Moreover u is unique and is defined by the formula $u(re^{i\theta}) = \frac{1}{2\pi} \int_{-\pi}^{\pi} P_r(\theta - t) f(e^{it})\, dt$ for $0 \le r < 1$, $0 \le \theta \le 2\pi$. **15**

OR

(B) State and prove Hadamard's factorizations theorems. **15**
