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Reg. No. : ....

Name : ....

Fourth Semester M.Sc. Degree Examination, July 2018
Branch : MATHEMATICS
MM 241 – Complex Analysis – II
(2011 Admission Onwards)

Time : 3 Hours Max. Marks : 75

Instructions : *Answer* ***either*** *Part –* ***A*** *or Part –* ***B*** *of* ***each*** *question.*
All *questions carry* ***equal*** *marks.*

I. A) a) Let the metric ρ be defined by $\rho(f, g) = \sum_{n=1}^{\infty} \frac{\rho_n(f, g)}{2^n (1+\rho_n(f, g))}$. If $\in > 0$ is given then prove that there is a $\delta > 0$ and a compact set $K \subset G$ such that for f and g in $C(G,\Omega), \sup\{d(f(z), g(z)) : z \in K\} < \delta \Rightarrow \rho(f, g) < \in$.

Conversely, if $\delta > 0$ and a compact set K are given, then prove that there is an $\in > 0$ such that for f and g in $C(G,\Omega), \rho(f, g) < \in \Rightarrow \sup\{d(f(z), g(z)) : z \in K\} < \delta$. 7

b) State and prove Arzela-Ascoli Theorem. 8

OR

B) a) If $\{f_n\}$ is a sequence in H(G) and f belongs to C(G, C) such that $f_n \to f$ then prove that f is analytic and $f_n^{(k)} \to f^{(k)}$ for each $k \geq 1$. 7

b) Prove that a family $\mathcal{F}$ in H(G) is normal iff $\mathcal{F}$ is locally bounded. 8

II. A) Let G be a region and let $\{a_j\}$ be a sequence of distinct points in G with no limit point in G; and let $\{m_j\}$ be a sequence of integers. Prove that there is an analytic function f defined on G whose only zeros are at the points a_j; furthermore, a_j is a zero of f of multiplicity m_j. 15

OR

B) State and prove Bohr-Mollerup theorem. 15

P.T.O.

III. A) a) For Re z > 1, prove that $\zeta(z)\Gamma z = \int_0^\infty (e^t - 1)^{-1} t^{z-1} dt$ 7

b) Prove that $\zeta(z) = 2(2\pi)^{z-1}\Gamma(1-z)\zeta(z)\mathrm{Sin}\left(\frac{\pi z}{2}\right)$, for $-1 < \mathrm{Re}z < 0$. 8

OR

B) Let K be a compact subset of the region G; then prove that there are straight line segments $\gamma_1, \gamma_2, \gamma_n$ in G – K such that for every function f in H(G), $f(z) = \sum_{k=1}^{n} \frac{1}{2\pi i} \int_{\gamma_k} \frac{f(w)}{w-z} dw$, for all $z \in K$. The line segments form a finite number of closed polygons. 15

IV. A) a) Let G be a region such that $G = G^*$. If $f : G_+ \cup G_0 \to C$ is a continuous function which is analytic on G_+ and if f(x) is real for x in G_0, then prove that there is an analytic function $g : G \to C$ such that $g(z) = f(z)$ for Z in $G_+ \cup G_0$. 8

b) Let $\gamma : [0,1] \to C$ be a path from a to b and let $\{(f_t, D_t); 0 \le t \le 1\}$ and $\{(g_t, B_t) : 0 \le t \le 1\}$ be analytic continuations along γ such that $[f_0]_a = [g_0]_a$. Then prove that $[f_1]_b = [g_1]_b$. 7

OR

B) a) Let $\gamma : [0,1] \to C$ be a path and let $\{(f_t, D_t) : 0 \le t \le 1\}$ be an analytic continuation along γ. For $0 \le t \le 1$, let R(t) be the radius of convergence of the power series expansion of f_t about $z = \gamma(t)$. Then prove that either $R(t) \equiv \infty$ or $R : [0,1] \to (0, \infty)$ is continuous. 6

b) State and prove Monodromy Theorem. 9

V. A) a) State and prove first vertion of Maximum principle. 6

b) Let G be a region. Prove that the metric space Har(G) is complete. Also, if $\{u_n\}$ is a sequence in Har(G) such that $u_1 \le u_2 \le \ldots$ then prove that either $u_n(z) \to \infty$ uniformly on compact subsets of G or $\{u_n\}$ converges in Har(G) to a harmonic function. 9

OR

B) a) Let f be an entire function of genus μ. Prove that for each positive number α there is a number γ_0 such that for $|z| > \gamma_0, |f(z)| < \exp(\alpha |z|^{\mu+1})$. 11

b) Let f be an entire function of finite order. Prove that f assumes each complex number with one possible exception. 4