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M – 1557

Reg. No. : ....

Name : ....

Fifth Semester B.Sc. Degree Examination, December 2021

First Degree Programme Under CBCSS

Statistics

Core Course VI

ST 1542 — ESTIMATION

(2014, 2016 – 2017 Admission)

Time : 3 Hours

Max. Marks : 80

SECTION – A

Answer **all** questions. **Each** question carries **1** mark.

1. Define :

   (a) Parameter

   (b) Statistic.

2. Define most efficient estimator.

3. Give sufficient conditions for consistency.

4. Give 95% confidence limits for the variance of $N(\mu, \sigma)$, when μ is known.

5. State invariance property of sufficient estimator.

6. What is minimum variance bound estimator.

7. State any two properties of maximum likelihood estimator.

P.T.O.

8. Define moment estimator.

9. Define quadratic loss function.

10. Define risk function.

(10 × 1 = 10 Marks)

SECTION – B

Answer **any eight** questions. **Each** carries **2** marks.

11. Let $X_1, X_2, X_3, \ldots X_n$ is a random sample drawn from $N(\mu, 1)$. Obtain an unbiased estimator for $\mu^2 + 1$.

12. If T is consistent estimator for θ, then show that T^2 is consistent for θ^2.

13. Find the moment estimator for the parameter θ in Poisson distribution with density function, $p(x,\theta) = \frac{e^{-\theta}\theta^x}{x!}$, $x = 0, 1, 2, \ldots, \theta > 0$.

14. Describe estimation of parameters using maximum likelihood estimation.

15. Derive 95% confidence interval for the difference of population proportions for large samples.

16. Check whether Minimum Variance Bound (MVB) estimator exists for the parameter θ in the Cauchy population with pdf $f(x,\theta) = \frac{1}{\pi}\frac{1}{1+(x-\theta)}$, $-\infty < x, \theta < \infty$.

17. How is Cramer-Rao inequality useful in obtaining MVUE?

18. Let $X_1, X_2, X_3, \ldots, X_n$ is a random sample drawn from a distribution with pdf $f(x,\theta) = e^{-(x-\theta)}$, $x \geq \theta$, $-\infty < \theta < \infty$. Obtain sufficient statistic for θ.

19. Let X be random variable with probability density function, $f(x,\theta) = (1+\theta)x^{\theta}$, $0 < x < 1$. Find MLE of θ based on a random sample of n observations.

20. Describe the estimation of parameters using method of least squares.

21. Describe Baye's estimation.

22. What is 0 – 1 loss function? Give a situation where it is employed.

(8 × 2 = 16 Marks)

SECTION – C

Answer **any six** questions. **Each** carries **4** marks.

23. What do you meant by point estimation? Give an example of an estimator

    (a) which is consistent but not unbiased,

    (b) which is unbiased but not consistent.

24. Let $X_1, X_2, X_3, ..., X_n$ be a random sample drawn from $N(\mu, \sigma^2)$ then obtain the maximum likelihood estimators for μ and σ^2.

25. Let X_1, X_2, X_3, X_4 be is a random sample of size 4 drawn from $N(\mu, \sigma^2)$. Let $i_1 = \dfrac{X_1 + X_2 + X_3 + X_4}{4}$, $t_2 = \dfrac{X_1 + 3X_2 + 2X_3 + X_4}{7}$, find efficiency of t_2 relative to t_1. Which is relatively more efficient estimator?

26. Describe the method of moments for estimating the parameters. What are the properties of the estimates obtained by this method?

27. Derive $100(1-\alpha)\%$ confidence interval for $\mu_1 - \mu_2$ if sample sizes n_1 and n_2 are large.

28. Let $X_1, X_2, X_3, ..., X_n$ be a random sample drawn from $N(\mu, 1)$. Obtain the MVUE of μ.

29. Define sufficient estimator. Let x_1, x_2 be i.i.d. $P(\lambda)$ random variables then show that $x_1 + 2x_2$ is not sufficient for λ.

30. $X_1, X_2, ..., X_n$ is a random sample from a population with pdf $f(x) = \dfrac{1}{\theta}$, $0 < x < \theta$. Show that $X_{(n)} = \max(X_1, X_2, ..., X_n)$ is sufficient for θ and $\dfrac{(n+1)}{n} X_{(n)}$ is an unbiased estimator for θ.

31. Discuss the meaning and calculation of the posterior distribution in Bayesian analysis. Give an example to illustrate the procedure.

(6 × 4 = 24 Marks)

SECTION – D

Answer **any two** questions. **Each** carries **15** marks.

32. (a) Prove that in sampling from $N(\mu, \sigma)$ population, the sample mean is a consistent estimator for μ.

    (b) Prove that for Cauchy sample mean is not consistent estimator but sample median is consistent for population mean.

33. Derive confidence interval for population mean when

    (a) σ known

    (b) σ unknown.

34. (a) State Cramer-Rao inequality and describe its regularity conditions,

    (b) A random sample $X_1, X_2, ..., X_n$ is taken from a uniform population with pdf, $f(x) = 1,\ \theta - \frac{1}{2} \le x_i \le \theta + \frac{1}{2}, -\infty < \theta < \infty$. Obtain the MLE for θ.

35. The sample values from population with pdf $f(x) = \theta x^{\theta - 1},\ 0 < x < 1,\ \theta > 0$, are given below :

    0.46, 0.38, 0.61, 0.82, 0.59, 0.33, 0.72, 0.44, 0.59, 0.60.

    (a) Derive moment estimate of θ

    (b) Compute the moment estimate of the parameter θ for the given data.

(2 × 15 = 30 Marks)