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Reg. No. : ....

Name : ....

Third Semester B.Sc. Degree Examination, March 2022

First Degree Programme under CBCSS

Mathematics

MM 1341 — ELEMENTARY NUMBER THEORY AND CALCULUS – I

(2018 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions. **Each** question carries **1** mark.

1. Find five consecutive integers that are composites.
2. State the prime number theorem.
3. If p is a prime and if $p \mid ab$, then prove that $p \mid a$ or $p \mid b$.
4. Express $3ABC_{sixteen}$ in *base ten*.
5. If $r(t) = t^2 i + e^t j - (2\cos \pi t)k$, compute $\lim_{t \to o} r(t)$.
6. Define the escape speed.
7. Determine whether the vector – valued function $r(t) = t^2 i + t^3 j$ is smooth.
8. State the extreme – value theorem.
9. Compute $\frac{dy}{dx}$ given that $x^3 + y^2 x - 3 = 0$.
10. Define the total differential of $w = f(x, y, z)$ at (x_0, y_0, z_0).

(10 × 1 = 10 Marks)

P.T.O.

SECTION – B

Answer any **eight** questions. **Each** question carries **2** marks.

11. Find the primes such that their digits in the decimal values alternate between 0s and 1s, beginning with and ending in 1.

12. Verify whether the LDEs $12x+18y=30$ and $6x+8y=25$ are solvable.

13. If $a\mid c$ and $b\mid c$, can we say that $ab\mid c$. Justify your answer.

14. Find the number of positive integers ≤ 3000 and divisible by 3, 5, or 7.

15. Estimate $\int_0^1 r(t)dt$, where $r(t)=2ti+t^2j-(\sin\pi t)k$.

16. Write the formulas for acceleration and speed in 3 – space.

17. Find $T(s)$ by parameterizing the circle $r=a\cos i+a\sin t\, j$, $0\leq t\leq 2\pi$, of radius a with counter clockwise orientation and centered at the origin.

18. Find the arc length of that portion of the circular helix $x=\cos t$, $y=\sin t$, $z=t$ from $t=0$ to $t=\pi$.

19. Determine maximum value of a directional derivative of $f(x,y)=x^2e^y$ at (–2, 0) and the unit vector in the direction in which the maximum value occurs.

20. Consider the sphere $x^2+y^2+z^2=1$. Evaluate $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ at $\left(\frac{2}{3},\frac{1}{3},\frac{2}{3}\right)$.

21. Compute $\lim_{(x,y)\to(0,0)}\left(x^2+y^2\right)\ln\left(x^2+y^2\right)$.

22. Verify : If $F(x,y,z)=2z^3-3\left(x^2+y^2\right)z$, then $F_{xx}+F_{yy}+F_{zz}=0$.

(8 × 2 = 16 Marks)

SECTION – C

Answer any **six** questions. **Each** question carries **4** marks.

23. Show that there are infinitely many primes of the form $4n+3$.

24. Find the number of trailing zeros in 234!.

25. Let a and b be positive integers. Derive a relationship between (a, b) and $[a, b]$. Also verify it for the integers 18 and 24.

26. Let $r_1(t) = (\tan^{-1} t)i + (\sin t)j + t^2 k$ and $r_2(t) = (t^2 - t)i + (2t-2)j + (\ln t)k$. The graphs of $r_1(t)$ and $r_2(t)$ intersect at the origin. Find the degree measure of the acute angle between the tangent lines to the graphs of $r_1(t)$ and $r_2(t)$ at the origin.

27. Find $r(t)$ given that $r'(t) = \langle 3, 2t \rangle$ and $r(1) = \langle 2, 5 \rangle$.

28. Compute the second – order partial derivatives of $f(x,y) = x^2y^3 + x^4y$.

29. For the function $f(x,y) = -\frac{xy}{x^2+y^2}$ estimate the limit of $f(x,y)$ as $(x,y) \to (0,0)$ along

    (a) $x - axis$

    (b) $y - axis$

    (c) the line $y = x$

    (d) the parabola $y = x^2$.

30. Given that $z = e^{xy}, x = 2u + v, y = \frac{u}{v}$, compute $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$.

31. Derive the parametric equations of the tangent line to the curve of intersection of the paraboloid $z = x^2 + y^2$ and the ellipsoid $3x^2 + 2y^2 + z^2 = 9$ at the point (1, 1, 2).

(6 × 4 = 24 Marks)

SECTION – D

Answer any **two** questions. **Each** question carries **15** marks.

32. (a) A six – digit positive integer is cut up in the middle into two three – digit numbers. If the square of their sum yields the original, find the number.

    (b) Solve the LDE $1076x + 2076y = 3076$ by Euler's method.

33. (a) A geosynchronous orbit for a satellite is a circular orbit about the equator of the Earth in which the satellite stays fixed over a point on the equator. Use the fact that the Earth makes one revolution about its axis every 24 hours to find the altitude in miles of a communications satellite in geosynchronous orbit. Assume the earth to be a sphere of radius 4000 miles.

    (b) In a projectile motion, derive the position function of the object in terms of its initial position and velocity.

34. Find the absolute maximum and minimum values of $f(x, y) = 3xy - 6x - 3y + 7$ on the closed triangular region R with vertices (0, 0), (3, 0) and (0,5).

35. Use Lagrange multipliers to determine the dimensions of a rectangular box, open at the top, having a volume of $32\ ft^3$, and requiring the least amount of material for its construction.

(2 × 15 = 30 Marks)