

Reg. No. :

Name :

Third Semester B.Sc. Degree Examination, March 2022

First Degree Programme under CBCSS

Statistics

Complementary Course for Mathematics

ST 1331.1 : STATISTICAL DISTRIBUTIONS

(2019 & 2020 Admission)

Time : 3 Hours

Max. Marks : 80

(Use of Calculator and Statistical table is permitted)

SECTION – A

Answer **all** questions. Each question carries **1** mark.

1. What is the mode of binomial distribution if $n = 10$ and $p = 0.4$?
2. The continuous distribution which satisfies the lack of memory property is _____.
3. Which type of convergence does the central limit theorem use?
4. What is the standard error of sample mean?
5. Write down the variance of chi-square distribution with 10 degrees of freedom.
6. The square of standard normal variate is called _____.
7. If X follows normal distribution with mean 30, what is $P(X < 30)$?
8. Name the probability distribution for which the first three moments are equal.
9. Which distribution is useful to estimate the number of fish in a lake?
10. What is the skewness for a normal distribution?

(10 × 1 = 10 Marks)

P.T.O.

SECTION – B

Answer **any eight** questions. Each question carries **2** marks.

11. Define Bernoulli trial, Bernoulli random variable and its distribution.
12. Write down the probability density function of normal distribution.
13. If a random variable X follows Poisson distribution such that $P(X = 1) = P(X = 2)$. Find the mean of the distribution.
14. State Lindberg-Levy form of central limit theorem.
15. Write down the probability density function of t distribution with n degrees of freedom.
16. Distinguish between parameter and statistic with examples.
17. Define standard normal variate. For a normal random variable X with mean 24 and variance 100, what is the value of standard normal variate corresponds to $X = 30$?
18. Distinguish between Fishers t statistic and student's t statistic.
19. Explain the concept of convergence in probability.
20. Define beta distribution of first kind.
21. If X has a uniform distribution in $[0, 1]$, show that $-2\log_e X$ follows exponential distribution.
22. The mean and variance of binomial distribution are 3 and 2 respectively. Find the probability that the variate takes the value zero.
23. State the additive property of Poisson distribution.
24. Define sampling distribution. If the parent population is normal, what is the distribution of sample mean?
25. State the conditions for weak law of large numbers to hold.
26. Find the distribution function of exponential distribution with mean $\frac{1}{\theta}$.

(8 × 2 = 16 Marks)

SECTION – C

Answer **any six** questions. Each question carries **4** marks.

27. Find the variance of binomial distribution with parameters n and p .
28. State and prove lack of memory property of geometric distribution.
29. Assume that half of the population is vegetarian so that the chance of an individual being a vegetarian is $\frac{1}{2}$. Assuming that 100 investigators take sample of 10 individuals to see whether they are vegetarians. How many investigators would you expect to report three people or less were vegetarians?
30. Define Poisson distribution. What are its mean and variance? Explain the importance of Poisson distribution in real situations.
31. A set of examinations marks is approximately normally distributed with a mean of 75 and standard deviation of 5. If the top 5% of the students is grade A and the bottom 25% get grade D. What marks is the lowest A and what marks the highest D?
32. If X and Y are independent exponential random variables, show that $\min(X, Y)$ has an exponential distribution.
33. Let X and Y are independent binomial variables with respective parameters $n = 3, p = 1/3$ and $n = 5, p = 1/3$. Find $P(X + Y \geq 1)$.
34. Find the characteristic function of chi-square distribution with n df.
35. A symmetric die is thrown 600 times. Find the lower bound for the probability of getting 80 to 120 sixes.
36. Derive the moment generating function of gamma distribution with one parameter and establish the additive property.
37. Suppose an average of one house in 1000 in a certain district has a fire during a year. If there are 2000 houses in that district, what is the probability that exactly 5 houses will have a fire during the year?
38. Let the random variables X_i for $i = 1, 2, \dots$, assume values i and $-i$ with equal probabilities. Examine whether weak law of large numbers holds for the sequence $\{X_i\}$.

(6 × 4 = 24 Marks)

SECTION – D

Answer **any two** questions. Each question carries **15** marks.

39. Derive the recurrence relation for the moments of binomial distribution and hence find its first four central moments, coefficient of skewness and coefficient of kurtosis. Explain the nature of skewness and kurtosis for the distribution.
(12 + 3 = 15)
40. (a) State the important properties of normal distribution.
(b) Find the moment generating function of normal distribution.
(c) Show that all odd order central moments of normal distribution are zero.
(6 + 6 + 3 = 15)
41. (a) Define hypergeometric distribution. Obtain the mean and variance of the distribution.
(b) If X and Y are independent and identically distributed random variables having geometric distributions, show that conditional distribution of X given $X + Y$ is uniform.
(10 + 5 = 15)
42. (a) Derive the recurrence formula for the probabilities of Poisson distribution.
(b) A systematic sample of 100 pages was taken from a dictionary and the observed frequency distribution of foreign words per page was found to be as follows :
- | | | | | | | | |
|-------------------------------|----|----|----|---|---|---|---|
| No. of foreign words per page | 0 | 1 | 2 | 3 | 4 | 5 | 6 |
| Frequency | 48 | 27 | 12 | 7 | 4 | 1 | 1 |
- Fit Poisson distribution and calculate the expected frequencies.
(2 + 13 = 15)
43. (a) Define exponential distribution. Derive its moment generating function and hence or otherwise find the mean and variance of the distribution.
(b) Obtain the mean and variance of beta distribution.
(8 + 7 = 15)
44. (a) Define F statistic and its distribution. Give any two applications of F distribution.
(b) What are the skewness for chi-square and t distributions?
(c) Point out any two applications of chi-square distribution in statistical theory.
(d) State and establish the relationship between t and F distribution.
(5 + 2 + 2 + 6 = 15)

(2 × 15 = 30 Marks)