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Reg. No. : ....

Name : ....

Third Semester B.Sc. Degree Examination, March 2022

First Degree Programme under CBCSS

Mathematics

MM 1341 : ELEMENTARY NUMBER THEORY AND CALCULUS — I

(2019 & 2020 Admission)

Time : 3 Hours Max. Marks : 80

PART – A

Answer **all** questions. Each question carries **1** mark.

1. Find five consecutive integers that are composites.

2. State the prime number theorem.

3. If p is a prime and if $p \mid ab$, then prove that $p \mid a$ or $p \mid b$.

4. Express $3ABC_{sixteen}$ in base ten.

5. If $r(t) = t^2 i + e^t j - (2\cos \pi t)k$, compute $\lim_{t\to 0} r(t)$.

6. Define the escape speed.

7. Determine whether the vector-valued function $r(t) = t^2 i + t^3 j$ is smooth.

P.T.O.

8. State the extreme-value theorem.

9. Compute $\frac{dy}{dx}$ given that $x^3 + y^2 x - 3 = 0$.

10. Define the total differential of $w = f(x, y, z)$ at (x_0, y_0, z_0).

(10 × 1 = 10 Marks)

PART – B

Answer **any eight** questions. Each question carries **2** marks.

11. Find the primes such that their digits in the decimal values alternate between 0s and 1s, beginning with the ending in 1.

12. Show that every integer $n \geq 2$ has a prime factor.

13. Verify whether the LDEs $12x + 18y = 30$ and $6x + 8y = 25$ are solvable.

14. If $a \mid c$ and $b \mid c$, can we say that $ab \mid c$? Justify your answer.

15. Find the number of positive integers ≤ 3000 and divisible by 3, 5, or 7.

16. Show that 111 cannot be a square in any base.

17. State any two rules of differentiation of vector-valued functions.

18. Estimate : $\int_0^1 r(t)\, dt$, where $r(t) = 2ti + t^2 j - (\sin \pi t)\, k$.

19. Write the formulas for acceleration and speed in 3-space.

20. Find $T(s)$ by parameterizing the circle $r = a\cos t\, i + a\sin t\, j, 0 \le t \le 2\pi$, of radius a with counter clockwise orientation and centered at the origin.

21. Find the arc length of that portion of the circular helix $x = \cos t, y = \sin t, \ z = t$ from $t = 0$ to $t = \pi$.

22. Determine maximum value of a directional derivative of $f(x, y) = x^2 e^y$ at $(-2, 0)$ and the unit vector in the direction in which the maximum value occurs.

23. Consider the sphere $x^2 + y^2 + z^2 = 1$. Evaluate $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ at $\left(\frac{2}{3}, \frac{1}{3}, \frac{2}{3}\right)$.

24. Compute $\lim_{(x, y) \to (0, 0)} (x^2 + y^2) \ln(x^2 + y^2)$.

25. Verify : If $F(x, y, z) = 2z^3 - 3(x^2 + y^2)z$, then $F_{xx} + F_{yy} + F_{zz} = 0$.

26. State the second partials test.

(8 × 2 = 16 Marks)

PART – C

Answer **any six** questions. Each question carries **4** marks.

27. Show that there are infinitely many primes of the form $4n + 3$.

28. Find the number of trailing zeros in 234!.

29. Let b be an integer ≥ 2. Suppose $b + 1$ integers are randomly selected. Prove that the difference of two of them is divisible by b.

30. Let a and b be positive integers. Derive a relationship between (a, b) and $[a, b]$. Also verify it for the integers 18 and 24.

31. Let $r_1(t) = (\tan^{-1} t)i + (\sin t)j + t^2k$ and $r_2(t) = (t^2 - t)i + (2t - 2)j + (\ln t)k$. The graphs of $r_1(t)$ and $r_2(t)$ intersect at the origin. Find the degree measure of the acute angle between the tangent lines to the graphs of $r_1(t)$ and $r_2(t)$ at the origin.

32. Find $r(t)$ given that $r'(t) = \langle 3, 2t \rangle$ and $r(1) = \langle 2, 5 \rangle$.

33. Show that the trajectory of a projectile is a parabolic path.

34. Derive Kepler's second law.

35. Compute the second-order partial derivatives of $f(x, y) = x^2y^3 + x^4y$.

36. For the function $f(x, y) = -\dfrac{xy}{x^2 + y^2}$, estimate the limit of $f(x, y)$ as $(x, y) \to (0, 0)$ along.

(a) x-axis

(b) y-axis

(c) the line $y = x$

(d) the parabola $y = x^2$.

37. Given that $z = e^{xy}$, $x = 2u + v$, $y = \dfrac{u}{v}$, compute $\dfrac{\partial z}{\partial u}$ and $\dfrac{\partial z}{\partial v}$.

38. Derive the parametric equations of the tangent line to the curve of intersection of the paraboloid $z = x^2 + y^2$ and the ellipsoid $3x^2 + 2y^2 + z^2 = 9$ at the point (1, 1, 2).

(6 × 4 = 24 Marks)

PART – D

Answer **any two** questions. Each question carries **15** marks.

39. (a) A six-digit positive integer is cut up in the middle into two three-digit numbers. If the square of their sum yields the original, find the number.

    (b) Solve the LDE $1076x + 2076y = 3076$ by Euler's method.

40. (a) State and prove the fundamental theorem of arithmetic.

    (b) Explain the Euclidean algorithm and evaluate (4076, 1024).

41. (a) A geosynchronous orbit for a satellite is a circular orbit about the equator of the Earth in which the satellite stays fixed over a point on the equator. Use the fact that the Earth makes one revolution about its axis every 24 hours to find the altitude in miles of a communications satellite in geosynchronous orbit. Assume the earth to be a sphere of radius 4000 miles.

    (b) In a projectile motion, derive the position function of the object in terms of its initial position and velocity.

42. (a) A particle moves through 3-space in such a way that its velocity is $v(t) = i + tj + t^2k$. Find the coordinates of the particle at time $t = 1$ given that the particle is at the point (–1, 2, 4) at time $t = 0$.

    (b) Find $k(t)$ for the circular helix $x = a\cos t, y = a\sin t, z = ct$ where $a > 0$.

43. Find the absolute maximum and minimum values of $f(x, y) = 3xy - 6x - 3y + 7$ on the closed triangular region R with vertices (0, 0), (3, 0) and (0, 5).

44. Use Lagrange multipliers to determine the dimensions of a rectangular box, open at the top, having a volume of $32 ft^3$, and requiring the least amount of material for its construction.

(2 × 15 = 30 Marks)
