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Reg. No. : ....

Name : ....

Fourth Semester B.Sc. Degree Examination, July 2019

First Degree Programme under CBCSS

Statistics

Core Course III

ST 1441 : PROBABILITY AND DISTRIBUTION – II

(2013 Admission onwards)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions. Each question carries 1 mark. :

1. If X is a random variable with variance 2, find the variance of $y = 2 - x$.

2. If X and Y are independent random variables such that $E(X) = E(Y) = 0$ and $\text{var}(X) = \text{var}(Y) = 1$, find $E[(X - Y)^2]$.

3. Give an example of a distribution whose expectation does not exist.

4. Define the probability generating function of a discrete random variable.

5. If X is a random variable with moment generating function $M_X(t) = e^{t^2}$, find the moment generating function of $Y = 2x - 3$.

6. Define Bernoulli distribution.

7. What do you mean by additive property of a distribution?

P.T.O.

8. If $X \sim$ binomial (n, p) with mean 6 and variance 2.4. Find n and p.

9. Define Poisson distribution.

10. Define Exponential distribution.

(10 × 1 = 10 Marks)

SECTION – B

Answer **any eight** questions. Each question carries **2** marks.

11. If the conditional distribution of X given $Y = y$ is $f(x \mid y) = \begin{cases} \frac{1}{(y+1)} e^{-x/(y+1)}, & x > 0 \\ 0, & elsewhere, \end{cases}$ find $E(X \mid Y)$, if Y is a non-negative random variable.

12. If the moment generating function of X is given by $M_X(t) = 0.2\, e^{3t} + 0.4 e^{-t} + 0.4 e^{2t}$, find the distribution of X.

13. Distinguish between correlation and regression.

14. Find the cumulant generating function of a Poisson variate with mean λ.

15. State and prove the additive property of Poisson distribution.

16. Define multinomial distribution and show that it is generalisation of binomial distribution.

17. If the moment generating function of a random variable is $M_X(t) = (0.2 + 0.8 e^t)^5$, find the mean and variance of X.

18. Derive the mean and variance of continuous uniform distribution over (–1, 1).

19. If $X \sim N(\mu, \sigma^2)$, find the distribution of $Y = 2 - 3X$.

20. Define geometric distribution and derive its mean.

21. Define type-I and beta distribution and find its mean and variance.

22. Find the distribution function corresponding to the following triangular distribution, with density function $f(x) = \begin{cases} 1 - |x|, & -1 < x < 1 \\ 0, & elsewhere. \end{cases}$

(8 × 2 = 16 Marks)

SECTION – C

Answer any **six** questions. Each question carries **4** marks.

23. Derive the moment generating function of the normal distribution $N(\mu, \sigma^2)$.

24. If X is a continuous random variable with distribution $F(x)$, show that $Y = F(x) \sim U(0, 1)$.

25. Define hyper-geometric distribution and derive its mean and variance.

26. State and prove multiplication theorem on expectation.

27. State and prove the lack of memory property of geometric distribution.

28. If $X_1, X_2, ..., X_n$ is a random sample from $N(\mu, \sigma^2)$, derive the distribution of $\overline{X} = (X_1 + ... + X_n)/n$.

29. State and prove the recurrence relation for the moments of a binomial distribution.

30. Find the mode of a Poisson distribution.

31. Define discrete uniform distribution over the set $\{1, 2, ..., n\}$ and derive its mean and variance.

(6 × 4 = 24 Marks)

SECTION – D

Answer any **two** questions. Each question carries **15** marks. :

32. (a) State and prove Cauchy-Schwartz inequality.

    (b) Using Cauchy-Schwartz inequality, show that correlation coefficient lies on [–1, 1]

33. Derive the central moments of $X \sim N(\mu, \sigma^2)$ and hence show that $\beta_1 = 0$ and $\beta_2 = 3$.

34. (a) Define triangular distribution and derive its distribution function, expectation and variance.

(b) Define type-III beta distribution and establish its relation with type-I beta distribution.

35. A bivariate random variable (X, Y) has the following joint density

$$f(x, y) = \begin{cases} \frac{2}{3}(x+2y), & \text{for } 0 < x < 1, 0 < y < 1 \\ 0, & \text{elsewhere.} \end{cases}$$

Find the conditional mean and conditional variance of X given $Y = \frac{1}{2}$. Also verify whether $E\{E(X \mid Y) = E(X)$.

(2 × 15 = 30 Marks)