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Reg. No. : ....

Name : ....

Fourth Semester B.Sc. Degree Examination, July 2019

First Degree Programme under CBCSS

Complementary Course for Statistics

MM 1431.4 : MATHEMATICS IV (LINEAR ALGEBRA)

(2013 admission onwards)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the first **ten** questions are compulsory. They carry **1** mark each.

1. Define basis of a vector space.

2. State Cauchy-Schwarz inequality.

3. Find norm of u if the vector $u = (1, 0, 1)$.

4. What is the dimension of $\mathbb{R}^2$ over $\mathbb{R}$.

5. Find the rank of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.

6. If A is a 4×3 matrix, then maximum value of rank of A is ————.

7. IF rank of a matrix is 0, then the matrix is ————.

8. Find the matrix of the quadratic form $Q(x) = x_3^2 - 4x_1x_2 + 4x_1x_3$.

P.T.O.

9. Define a rotation from $\mathbb{R}^2$ to $\mathbb{R}^2$.

10. Does there exist a one-one linear transformation from $\mathbb{R}^3$ to $\mathbb{R}^2$.

(10 × 1 = 10 Marks)

SECTION – II

Answer **any eight** questions from among the questions 11 to 22. These questions carry **2** marks each.

11. Find k so that $u = (1, 1, k)$ and $v = (1, 2, -3)$ are orthogonal.

12. Check whether $\{(1, 2), (1, 3), (1, 0)\}$ is a basis for $\mathbb{R}^2$ over $\mathbb{R}$.

13. Prove that the set of vectors $\{(1, 2, 3), (2, 2, 4), (1, 0, 1)\}$ is linearly dependent.

14. Prove that $u = (1, 1, 1), v = (1, 2, 3)$ and $w = (1, 5, 8)$ are linearly independent vectors in $\mathbb{R}^3$.

15. Find the rank of matrix $\begin{bmatrix} 1 & 1 & 1 \\ 2 & 0 & 3 \\ 1 & 1 & 2 \end{bmatrix}$.

16. Show that the system of equations is consistent.

$$x + 2y = 5$$
$$3x + 5y = 13$$

17. Show that the matrix and its transpose have the same eigen values.

18. Find the eigen values of the matrix $\begin{bmatrix} 3 & 2 \\ -1 & 1 \end{bmatrix}$.

19. If $A = \begin{bmatrix} 1 & 2 \\ -1 & 3 \end{bmatrix}$, show that $A^2 - 4A + 5I = 0$.

20. Check whether the mapping $T(x, y) = (0, 0)$ is linear.

21. Find the matrix of the linear transformation $T(x, y) = (x+3y, x-5y)$ relative to the basis $\{(1, 0), (0, 1)\}$.

22. Prove that the identify transformation on $\mathbb{R}^n$ is linear. **(8 × 2 = 16 Marks)**

SECTION – III

Answer **any six** questions from among the questions **23** to **31**. These questions carry **4** marks each.

23. Write the vector $(1, -2, 5)$ as a linear combination of (1, 1, 1), (1, 2, 3) and (2, –1, 1).

24. Reducing to the echelon form find the rank of the matrix $\begin{bmatrix} 1 & -1 & 2 \\ 5 & 1 & 3 \\ 1 & 1 & 2 \end{bmatrix}$.

25. Solve the system of equations.

$$x+y-z=0$$
$$x-y+2z=0$$
$$3x+y=0$$

26. Solve the system of equations :

$$x+2y+z=3$$
$$2x+5y-z=-4$$
$$3x-2y-z=5$$

27. Check whether $T(x, y) = (x+y, x+2y)$ is one-one.

28. Find the eigen vectors of the matrix $\begin{bmatrix} 2 & 1 \\ 0 & 3 \end{bmatrix}$.

29. If $A = \begin{bmatrix} 2 & 2 & -5 \\ 3 & 7 & -15 \\ 1 & 2 & -4 \end{bmatrix}$, show that $A^3 - 5A^2 + 7A - 3I = 0$.

30. Find the matrix of the linear transformation $T(x, y) = (x + y, 2x - 5y)$ relative to the basis $\{(1, 1), (2, 1)\}$.

31. Make a change of variable $x = Py$, that transforms the quadratic form $x_1^2 + 10x_1x_2 + x_2^2$ into a quadratic form without cross product them. Give P and the new quadratic form. **(6 × 4 = 24 Marks)**

SECTION – IV

Answer **any two** questions from among the questions **32** to **35**. These questions carry **15** marks each.

32. (a) Check whether $W = \{(x, y, z) : x^2 + y^2 + z^2 \leq 1\}$ is a sub space of $\mathbb{R}^3$.

(b) Prove that a set of n vectors containing the zero vectors is linearly dependent.

33. Investigate for what values of λ, μ the system of equations

$x + y + z = 6, x - 2y + 3z = 10, x + 2y + \lambda z = \mu$ has (a) no solution (b) a unique solution (c) infinitely many solutions.

34. Find the eigen values and corresponding eigen vectors of the matrix $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$.

35. Prove that $A = \begin{bmatrix} 4 & 1 & -1 \\ 2 & 5 & -2 \\ 1 & 1 & 2 \end{bmatrix}$ is diagonalisable and find its diagonal form.

(2 × 15 = 30 Marks)
