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Reg. No. : ....

Name : ....

First Semester B.Sc. Degree Examination, June 2022

First Degree Programme under CBCSS

Mathematics

Complementary Course I for Chemistry and Polymer Chemistry

MM 1131.2 : MATHEMATICS I — CALCULUS WITH APPLICATIONS IN CHEMISTRY I

(2018 & 2019 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the **first ten** questions are compulsory. They carry **1** mark each.

1. Find the first derivative of $\cos 2x$.

2. Find the 1000^{th} derivative of e^x.

3. Define stationary point.

4. State Leibnitz's point.

5. State Demoivre's theorem.

6. Define argument of a complex number.

7. Find the complex conjugate of $2-2i$.

8. If $v = 3i - 4j$ is a velocity vector. Then find speed.

9. Define dot product.

10. Evaluate $\int x \sin x$.

(10 × 1 = 10 Marks)

SECTION – II

Answer **any eight** questions. These questions carry **2** marks each.

11. If $x = \sec t$ and $y = \tan t$, find $\frac{dy}{dx}$.

12. Find $\frac{dy}{dx}$, if $e^x - \sin y = x$.

13. Express $\frac{(x-iy)^2}{(x+iy)}$ in the form $a+bi$.

14. Find modulus of $6+8i$.

15. Find $\frac{d}{dx}(\cosh x)$.

16. Find the angle between two vectors a and b with magnitudes $\sqrt{3}$ and 2 respectively, and such that $a, b = \sqrt{6}$.

17. Find the value of p for which the vectors $3i + 2j + 9k$ and $i + pj + 3k$ are perpendicular.

18. Show that if $a = b + \lambda c$, for some scalar λ, then $a \times c = b \times c$.

19. Find the unit vector corresponding to the vector $i + j + k$.

20. Find the area of the parallelgram with sides $i + 2j + 3k$ and $4i + 5j + 6k$.

21. Evaluate $\int_0^{\infty} \frac{x}{(x^2+a^2)^2} dx$.

22. Find the mean value m of the function $f(x) = 3x^2 - 3$ between the limits $x = 0$ and $x = 1$.

(8 × 2 = 16 Marks)

SECTION – III

Answer **any six** questions. These questions carry **4** marks each.

23. Find the magnitude of radius of curvature at a point (x, y) on the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

24. Find the positions and stationary points of the function $f(x) = 3x^4 - 4x^3 - 8$.

25. Solve the hyperbolic equation $\cosh x - 5\sinh x - 5 = 0$.

26. Find the value of $(1+i)^i$.

27. Find an expression for $\cos^3\theta$ in terms of $\cos 3\theta$ and $\cos\theta$.

28. Evaluate $\int e^{ax} \cos bx\, dx$.

29. The vertices of triangle ABC have positive vectors a, b and c relative to some origin O. Find the position vector of the centroid G of the triangle.

30. A line is given by $r = a + \lambda b$, where $a = i + 2j + 3k$ and $b = 4i + 5j + 6k$. Find the coordinates of the point P at which the line intersects the plane $x + 2y + 3z = 6$.

31. Find the volume of a cone enclosed by the surface formed by rotating the curve $y = 2x$ about the x-axis the line between $x = 0$ and $x = h$.

(6 × 4 = 24 Marks)

SECTION – IV

Answer **any two** questions. These questions carry **15** marks each.

32. (a) Determine inequalities satisfied by $\ln x$ and $\sin x$ for suitable ranges of the real variable x.

    (b) Determine the constants a and b so that the curve $y = x^3 + ax^2 + bx$ has a stationary point inflection at the point $(3, -9)$.

33. (a) Express $\cosh^{-1} x$ in logarithmic form.

    (b) Evaluate $\frac{d}{dx}(\sinh^{-1} x)$.

34. (a) A point P divides a line segment AB in the ratio $\lambda : \mu$. If the position vectors of the points A and B are a and b respectively, find position vector of the point P.

    (b) Find the minimum distance from the point P with coordinates (1, 2, 1) to the line $r = a + \lambda b$, where $a = i + j + k$ and $b = 2i - j + 3k$.

35. (a) Using integration by parts, find a relationship between I_n and I_{n-1} where $I_n = \int_0^1 (1 - x^3)^n dx$ and n is any positive integer. Hence evaluate $I_2 = \int_0^1 (1 - x^3)^2 dx$.

    (b) Find the surface area of a cone formed by rotating about the x-axis the line $y = 2x$ between $x = 0$ and $x = h$.

(2 × 15 = 30 Marks)