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**Reg. No. : ..............................**

**Name : ..................................**

## First Semester B.Sc. Degree Examination, June 2022

## First Degree Programme under CBCSS

## Mathematics

## Complementary Course I for Chemistry and Polymer Chemistry

## MM 1131.2 : MATHEMATICS I – DIFFERENTIAL CALCULUS AND SEQUENCE AND SERIES

## (2021 Admission)



Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions.

1. Find $\frac{d}{dx}\left(\sqrt[3]{x}\right)$.

2. Compute the derivative of $\tan\left(x^2+1\right)$ with respect to $x$.

3. Find the inflection points, if any, of $f(x)=x^4$.

4. State the extreme-value theorem.

5. If $f(x, y)=\sqrt{y+1}+\ln\left(x^2-y\right)$, find $f(e, 0)$.

6. Write the one-dimensional wave equation.

7. When do we say that a function $f$ of two variables has an absolute maximum at $(x_0, y_0)$?

8. Show that $\lim_{n\to\infty}\sqrt[n]{n}=1$.

P.T.O.

9. Verify whether the series $\sum_{k=1}^{\infty} \frac{k}{k+1}$ converges.

10. Write the Bessel function $J_1(x)$ using sigma notation.

**(10 × 1 = 10 Marks)**

SECTION – B

Answer any **eight** questions.

11. Evaluate : $\lim_{x \to +\infty}\left(\sqrt{x^6+5}-x^3\right)$.

12. Compute : $\frac{ds}{dt}$ if $s=(1+t)\sqrt{t}$ .

13. Estimate $\frac{dy}{dx}$ if $y=\cos\left(x^3\right)$.

14. Use implicit differentiation to find $\frac{d^2y}{dx^2}$ if $4x^2-2y^2=9$.

15. Obtain the value of $\lim_{n \to \pi/2} \frac{1-\sin x}{\cos x}$.

16. Show that $f(x)=x^3$ has no relative extreme.

17. Write a procedure for finding absolute, extreme of a continuous function $f$ on a finite closed interval $[a, b]$.

18. State the mean value theorem.

19. Find $\frac{dz}{dx}$ and $\frac{\partial z}{\partial y}$ if $z=x^4\sin\left(xy^3\right)$.

20. State the chain rules for derivatives.

21. Given that $z=e^{xy}$, $x=2u+v$, $y=\frac{u}{v}$, compute $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$.

22. Consider the sphere $x^2+y^2+z^2=1$. Evaluate $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ at $\left(\frac{2}{3}, \frac{1}{3}, \frac{2}{3}\right)$.

23. Determine whether the sequence $\left\{\frac{n}{2n+1}\right\}_{n=1}^{+\infty}$ converges or diverges by examining the limit as $n \to +\infty$.

24. State the ratio test.

25. Using the root test check the convergence of the series $\sum_{k=2}^{\infty}\left(\frac{4k-5}{2k+1}\right)^k$.

26. Define the Taylor series for $f$ about $x = x_0$.

**(8 × 2 = 16 Marks)**

SECTION – C

Answer any **six** questions.

27. Compute : $\lim_{x\to 5}\frac{x^2-3x-10}{x^2-10x+25}$.

28. Find : $\frac{dy}{dx}$ if $y = \frac{\sin x}{1+\cos x}$.

29. Evaluate : $\frac{d}{dx}\left[\sin\sqrt{1+\cos x}\right]$.

30. Estimate : (i) $\lim_{x\to+\infty}\frac{x}{e^x}$ (ii) $\lim_{x\to 0+}\frac{\ln x}{\csc x}$.

31. Find : $\lim_{x\to 0}(1+\sin x)^{1/x}$.

32. Identify the intervals on which $f(x) = x^2 - 4x + 3$ is increasing and the intervals on which it is decreasing.

33. Find the second order partial derivatives of $f(x, y) = x^2y^3 + x^4y$.

34. Suppose $w = \sqrt{x^2+y^2+z^2}$, $x = \cos\theta$, $y = \sin\theta$, $z = \tan\theta$. Find $\frac{dw}{d\theta}$ when $\theta = \frac{\pi}{4}$.

35. Use appropriate forms of the chain rule to find $\frac{\partial w}{\partial \rho}$ and $\frac{\partial w}{\partial \theta}$ where $w = x^2 + y^2 - z^2$, $x = \rho\sin\phi\cos\theta$, $y = \rho\sin\phi\sin\theta$ and $z = \rho\cos\phi$.

36. Find the interval of convergence and radius of convergence of the series $\sum_{k=1}^{\infty}\frac{(x-5)^k}{k^2}$.

37. Use an $n^{th}$ Maclaurin polynomial for $e^x$ to approximate $e$ to five-decimal place accuracy.

38. Find the first four Taylor polynomials for $\ln x$ about $x = 2$.

**(6 × 4 = 24 Marks)**

## SECTION – D

Answer any **two** questions.

39. (a) Find $f''(\pi/4)$ if $f(x) = \sec x$.

    (b) On a sunny day, a 50 *ft* flagpole casts shadow that changes with the angle of elevation of the Sun. Let $s$ be the length of the shadow and $\theta$ the angle of elevation of the Sun. Find the rate at which the length of the shadow is changing with respect to $\theta$ when $\theta = 45°$. Express your answer in units of *feet/degree*.

    (c) Compute $\frac{d}{dx}\left[\ln\left(\frac{x^2 \sin x}{\sqrt{1+x}}\right)\right]$.

40. Sketch the graph of the equation $y = x^3 - 3x + 2$ and identify the locations of the intercepts, relative extrema, and inflection points.

41. (a) Find the slope of the sphere $x^2 + y^2 + z^2 = 1$ in the $y$-direction at the points $\left(\frac{2}{3}, \frac{1}{3}, \frac{2}{3}\right)$ and $\left(\frac{2}{3}, \frac{1}{3}, -\frac{2}{3}\right)$.

    (b) Describe the level surface of $f(x, y, z) = x^2 + y^2 + z^2$.

42. Use Lagrange multipliers to determine the dimensions of a rectangular box, open at the top, having a volume of 32 $ft^3$, and requiring the least amount of material for its construction.

43. (a) Use the comparison test to determine whether the following series converge or diverge :

    (i) $\sum_{k=1}^{\infty} \frac{1}{\sqrt{k} - \frac{1}{2}}$ (ii) $\sum_{k=1}^{\infty} \frac{1}{2k^2 + k}$

    (b) Prove that the series $\sum_{k=1}^{\infty} \frac{1}{k(k+1)}$ converges. Find the sum.

44. Find the Maclaurin series for

    (a) $e^x$ (b) $\sin x$ (c) $\cos x$ (d) $\frac{1}{1-x}$.

**(2 × 15 = 30 Marks)**