(Pages : 4) **G – 4031**

Reg. No. : ....

Name : ....

Fourth Semester B.Sc. Degree Examination, July 2019

First Degree Programme under CBCSS

Mathematics

Core Course – III

MM 1441 : METHODS OF ALGEBRA AND CALCULUS – II

(2013 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the first **ten** questions are compulsory. They carry **1** mark each.

1. Let $\mathbb{R}$ be a commutative ring without zero divisors. Then deg($f\,g$) is

2. State Root Theorem.

3. For what values of k in $\mathbb{Q}$, does x – k divide $x^3 - kx^2 - 2x + k + 3$.

4. State Primitive Element Theorem.

5. State Bezout's Identity.

6. Find $\lim_{(x,y)\to(0,0)} \dfrac{x^2 - xy}{\sqrt{x} - \sqrt{y}}$.

P.T.O.

7. If $z = x^2 + xy + 1$, find $\frac{\partial z}{\partial x}$.

8. If $u = x + y + 1$, find $\frac{\partial^2 u}{\partial x^6}$.

9. Evaluate $\int_0^1 \int_0^2 (x+3)\, dy\, dx$.

10. Define critical point of a function $f(x, y)$.

SECTION – II

Answer any **eight** questions from among the question 11 to 22. These questions carry **2** marks each.

11. Let R = $\mathbb{Z}/6\mathbb{Z}$ Find all units of *R*.

12. Find the remainder when $x^4 - 7x^2 + 3$ is divided by $x + 1$.

13. Prove that $\sum_{e/p-1} N(e) = p - 1$.

14. If *R* is an integral domain, show that *R*[*x*] is also an integral domain.

15. Find the level curves of the function $f(x, y) = 4x^2 + 9y^2$.

16. Find the domain of the function $f(x, y, z) = e^{xyz}$.

17. If $f(x, y) = x^2 + y \ln x$, find f_{xy}.

18. Find $\frac{dy}{dx}$ if $1 - x - y^2 - \sin(xy) = 0$.

19. If $w = xy + yz + zx, x = u + v, y = u - v$ and $z = uv$, find $\frac{\partial w}{\partial v}$.

20. Evaluate the double integral $\iint_R y^2 x\, dA$ over the rectangle

$$R = \{(x,y): -3 \le x \le 2, 0 \le y \le 1\}$$

21. Evaluate $\int_0^2 \int_{x^2}^{x} y^2 x dy\, dx$.

22. Evaluate $\iint_R xy\, dA$ over the region R enclosed between $y = \frac{1}{2}x, y = \sqrt{x}, x = 2$ and $x = 4$.

SECTION – III

Answer **any six** questions from among the question 23 to 31. These questions carry 4 marks **each**.

23. Find the quotient and remainder when $x^3 - 7x - 1$ is divided by $x - 2$ in $\mathbb{Q}[x]$.

24. State and prove Remainder Theorem.

25. Write $(x^2 + 3x + 1)^4$ in base $x + 2$.

26. Factor $x^6 + x^2 + x + 1$ into irreducible polynomials in $\mathbb{Z}/5\mathbb{Z}[x]$.

27. If $u = \ln(x^2 + y^2 + z^2)$, show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial z^2} = \frac{2}{x^2 + y^2 + z^2}$.

28. Obtain the extreme values of the following function $f(x,y) = x^2 + xy + y^2 + 3x - 3y + 4$.

29. Find the area of enclosed by the three-petaled rose $r = \sin x 3\theta$.

30. Evaluate $\iiint_G 12xy^2 z^3 dV$ over the rectangular box G defined by the inequalities $-1 \le x \le 2$, $0 \le y \le 3$, $0 \le z \le 2$.

31. Find the surface area of the portion of the surface $z = \sqrt{4 - x^2}$ that lies above the rectangle R in the xy – plane whose coordinates satisfy $0 \le x \le 1$ and $0 \le y \le 4$.

SECTION – IV

Answer **any two** questions from among the question 32 to 35. These questions carry **15** marks each.

32. (a) Using Euclid's algorithm find a greatest common divisor in $\mathbb{F}_3[x]$ of x^2+1 and x^5+1.

    (b) Prove that $N(p-1)=\phi(p-1)$.

33. (a) State and prove the Division Theorem for $\mathbb{F}[x]$, where $\mathbb{F}$ is a field.

    (b) Prove that for any $n, \sum_{d/n} \phi(d)=n$.

34. Find the points on the sphere $x^2+y^2+z^2=36$ that are closest to and farthest from the point (1, 2, 2).

35. (a) Find the area of the region R enclosed between the parabola $y=\frac{1}{2}x^2$ and the line $y=2x$.

    (b) Find the volume of the solid within the cylinder $x^2+y^2=9$ and between the planes z = 1 and $x+z=5$.
