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Reg. No. : ....

Name : ....

Fourth Semester B.Sc. Degree Examination, July 2019

First Degree Programme under CBCSS

Mathematics

Core Course – III

MM 1441 : ALGEBRA AND CALCULUS – II

(2014 Admission Onwards)

Time : 3 Hours Max. Marks : 80

UNIT – I

Answer **all** questions from this unit. Each question carries **1** mark.

1. Find $(x+1)(x^2+x+1)$ in $\mathbb{F}_2[x]$.

2. Find the quotient when x^3-7x-1 is divided by $x-2$.

3. Does $x-3$ divide x^4+x^3+x+4 in $\mathbb{Z}[x]$.

4. In $\mathbb{F}_3[x]$, find a greatest common divisor of x^2-x+4 and x^3+2x^2+3x+2.

5. The polynomial ax^2+bx+c is irreducible if and only if....

6. What is the natural domain of the function $f(x, y) = \ln(x^2-y^2)$?

7. Describe the level curves of the function $f(x, y) = y^2-x^2$.

P.T.O.

8. If $f(x,y,z) = x^3y^2z^4 + 2xy + z$, the find f_y.

9. Find $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^4 - 16y^4}{x^2 + 4y^2}$.

10. Evaluate $\int_1^3\int_1^2 (1 + 8\,xy)\,dy\,dx$.

(10 × 1 = 10 Marks)

UNIT – II

Answer **any eight** questions from this unit. Each question carries **2** marks.

11. Find another polynomial $q(x)$ with coefficients in $\mathbb{Z}/6\mathbb{Z}$ such that $q(x)$ is equal to $p(x) = [3]x + [4]x^3$ as functions on $\mathbb{Z}/6\mathbb{Z}$ but $p(x)$ and $q(x)$ are not equal as polynomials.

12. Let $R = \mathbb{Z}/4\mathbb{Z} = \{0, 1, 2, 3\}$. Show that $1 + 2x$ is a unit of $R(x)$.

13. Find all m so that the image of $x^3 + 3$ divides the image of $x^5 + x^3 + x^2 - 9$ in $\mathbb{Z}/m\mathbb{Z}\,[x]$.

14. State Bezout's identify.

15. Factorize $x^6 + x^4 + x$ is $\mathbb{Z}/2\mathbb{Z}\,[x]$.

16. Describe the level surfaces of $f(x,y,z) = x^2 + y^2 + z^2$.

17. Find $\lim\limits_{(x,y)\to(0,0)} \left(x^2 + y^2\right)\ln\left(x^2 + y^2\right)$.

18. Let $f(x,y) = x^2 + 5y^3$. Find the slope of the surface $z = f(x,y)$ in the x-direction at the point (1, –2).

19. Given that $x^3 + y^2x - 3 = 0$. Find $\dfrac{dy}{dx}$.

20. Evaluate $\iint_R xy\,dA$ over the region R enclosed between $y = \frac{1}{2}x, y = \sqrt{x}, x = 2$ and $x = 4$.

21. Find parametric equations for the portion of the right circular cylinder $x^2 + y^2 = 9$ for which $0 \le y \le 5$.

22. Evaluate $\int_0^2 \int_{-1}^{y^2} \int_{-1}^{z} yz\,dx\,dz\,dy$.

(8 × 2 = 16 Marks)

UNIT – III

Answer **any six** questions from this unit. Each question carries **4** marks.

23. Solve the equation $x^3 + 3x = 14$ by Ferro's method.

24. Find the g.c.d. of $x^5 + 1$ and $x^3 + 1$ in $\mathbb{Z}/2\mathbb{Z}$.

25. Prove that a non zero polynomial $f(x)$ of degree n in $F[x]$, F a field, has at most n distinct roots in F.

26. For any $e > 1$ dividing $p - 1$, if $N(e) > o$, then $N(e) = \phi(e)$. That is, $\phi(e)$ is the number of elements of U_p of order e?

27. Write $(x^2 + 3x + 1)^4$ in base $x + 2$.

28. At what rate is the volume of a box changing if its length is 8 m and increasing at 3 m/s, its width is 6 m increasing at 1 m/s and its height is 4 m and increasing at 1 m/s?

29. Locate all relative extrema and saddle points of $f(x, y) = 4xy - x^4 - y^4$.

30. The sphere of radius a is centered at the origin. Find the volume of the sphere.

31. Use a double integral to find the volume of the tetrahedron bounded by the co-ordinate planes and the plane $z = 4 - 4x - 2y$.

(6 × 4 = 24 Marks)

UNIT – IV

Answer **any two** questions from this unit. Each question carries **15** marks.

32. (a) Find a root of $y^4 + 2y^2 - y + 2$.

    (b) Factories $x^5 - x$ into irreducible polynomials in $\mathbb{Z}/5\mathbb{Z}\ [x]$.

33. State and prove the Division Theorem of $\mathbb{F}\ [x]$ where $\mathbb{F}$ is a field. Deduce the Remainder theorem.

34. (a) State and Constrained-Extremum Principle for two variables and one constraint.

    (b) Find the points on the sphere $x^2 + y^2 + z^2 = 36$ that are closest to and farthest from the point (1, 2, 2).

35. (a) Find an equation of the tangent plane to the parametric surface $x = uv$, $y = u, z = v^2$ at the point which corresponds to $(u, v) = (2, -1)$

    (b) Consider the sphere $x^2 + y^2 + z^2 = a^2$. Show that at each point on the sphere the tangent plane is perpendicular to the radius vector.

(2 × 15 = 30 Marks)