(Pages : 7) **K – 2431**

Reg. No. : ....

Name : ....

Third Semester B.Sc. Degree Examination, March 2021.

First Degree Programme under CBCSS

Mathematics

Complementary Course for Chemistry and Polymer Chemistry

MM 1331.2 – Mathematics – III – LINEAR ALGEBRA, PROBABILITY THEORY AND NUMERICAL METHODS

(2019 Admission Regular)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the **first ten** questions are compulsory. They carry **1** mark each.

1. Define equivalent matrices.

2. If A is an invertible matrix of order 3, then rank of A = ————

3. Trace of the matrix $\begin{bmatrix} 1 & -1 & 2 \\ 2 & -1 & 6 \\ 4 & -4 & 5 \end{bmatrix}$ is ————

4. Define wronskian of a set of functions.

5. If a three digit number is selected at random, what is the probability that the hundreds digit is 6?

6. What is probability density function?

P.T.O.

7. There are 10 questions on a test and you are to do 8 of them. In how many ways can you choose them?

8. When do the Poisson distribution gives a good approximation to the binomial distribution?

9. What is the order of convergence of the solution in Newton-Raphson method?

10. Give an example of a functions which is not analytically integrable over some range.

SECTION – II

Answer **any eight** question form among the questions 11 to 26. These questions carry **2** marks each

11. If $\bar{a} = 4\bar{i} - 3\bar{k}$ and $\bar{b} = -2i + 2j - \bar{k}$, find the

   (a) Scalar projection of $\bar{a}$ and $\bar{b}$

   (b) Scalar projection of $\bar{b}$ onto $\bar{a}$

12. Find the rank of the matrix $\begin{bmatrix} 1 & -1 & 2 & 3 \\ -2 & 2 & -1 & 0 \\ 4 & -4 & 5 & 6 \end{bmatrix}$.

13. If C is orthogonal and M is antisymmetric, show that the matrix $C^{-1}MC$ is antisymmetric.

14. Show that the functions $x, \sin x, 2x - 3\sin x$ are linearly dependent.

15. Find the equation of the line through $(3, 0, -5)$ and parallel to the line $\bar{r} = (2,1,-5) + (0,-3,1)t$.

16. Find the direction of the line of intersection of the planes $x - 2y + 3z = 4$ and $2x + y - z = 5$

17. Given a family of two children (assume boys and girls are equally likely), what is the probability that at least one is a girl? Given that the first two are girls, What is the probability that an expected third child will be a boy?

18. Two dice are thrown. What is the probability of being able to form a two-digit number greater than or equal to 42 with the two numbers on the dice?

19. Find the probability of getting 4 heads in 6 tosses of a fair coin.

20. An urn contains 10 black and 10 white balls. Find the probability of drawing two balls of the same colour.

21. Find the number of ways in which 15 balls can be put into 6 boxes in such a manner that the boxes contains 4, 3, 3, 2, 2, 1 balls respectively.

22. If a random variable x follows Poisson distribution such that $p(x=1)=p(x=2)$, then find the mean and variance of the distribution.

23. Find a red root of the equation $x^3+x^2-1=0$, lying between 0 and 1 by the method of rearrangement.

24. Using gauss elimination method, solve the equations

$$x+4y-3z=-5,$$
$$x+y-6z=-12,$$
$$3x-y-z=4.$$

25. Given a random number η uniformly distributed on (0,1), determine the function $\xi=\xi(\eta)$ that would generate a random number ξ distributed as 2ξ on $0\le\xi<1$.

26. Evaluate $\int_0^2 (x^2-3x+4)\,dx$ using trapezium rule with $h=0.5$.

SECTION – III

Answer **any six** questions from among the questions 27 to 38 then questions carry **4** marks each.

27. Determine the value of k such that the equations

$$x + ky + 3z = 0,$$
$$4x + 3y + kz = 0,$$
$$2x + y + 2z = 0$$

have a non-trival solution.

28. Find the equation of the plane through the points

$(-1, 1, 1)$, $(2, 3, 0)$ and $(0, 1, -2)$.

29. Find the angle between

(a) the space diagonals of a cube

(b) space diagonal and an edge

(c) a space diagonal and a diagonal of a face.

30. Find the dimension and a basis of the vector space spanned by the vectors (0, 1, 0, 6, -3), (1, 0, 1, 5, -2), (2, -1, 2, 4, 1) and (3, 0, 3, 15, -6).

31. What is the probability that a number n, $1 \leq n \leq 99$,

(a) is divisible by both 6 and 10?

(b) is divisible by either 6 or 10 or both?

32. A club consists of 50 members. In how many ways can a president, vice president, secretary and treasurer be chosen? In how many ways can a committee of 4 members be chosen?

33. A problem is chemistry is given to three students A, B, C whose chances of solving of are $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}$ respectively. What is the probability that the problem will be solved?

34. Suppose 3% of bolts made by a machine are defective, the defects occurring at random during production of bolts are packaged 50 per box, find

(a) exact probability and

(b) Poisson approximation to it, that a given box will contain 5 defectives.

35. Using Newton-Raphson procedure find, correct to three decimal places, the root nearest to 2 of the equation $x^4 + x^3 - 7x^2 - x + 5 = 0$.

36. Show that the iteration scheme

$x_{n+1} = \frac{1}{2}\left(x_n + \frac{x}{x_n}\right)$, for finding the square root of x has second order convergence. Using this evaluate $\sqrt{18}$ correct to four decimal places.

37. Use a Taylor series to solve the initial value problem $\frac{dy}{dx} + xy = 0$, $y(0) = 1$ evaluating $y(x)$ for $x = 0.0$ to 0.5 in steps of 0.1.

38. Apply gauss-seidal method to solve the equations

$$\begin{aligned} 5x + 2y + z &= 12 \\ x + 4y + 2z &= 15 \\ x + 2y + 5z &= 20 \end{aligned}$$

SECTION – IV

Answer **any two** questions from among the questions 39 to 44.
These questions carry **15** marks each.

39. (a) Let $M = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$. Find a matrix C such that $C^{-1}MC$ is a diagonal matrix.

(b) Find the equation relative to the principal axes of the quadric surface $6x^2 + 3y^2 + 3z^2 - 4xy + 4xz + 2yz = 60$. Also find the matrix of transformation.

40. Find the eigen values and corresponding independent eigen vectors of the matrix

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

41. Suppose that boxes of a certain kind of cereal have an average weight of 16 ounces and it is known that 70% of the boxes weigh within 1 ounce of the average. What is the probability that the box you buy weighs less than 14 ounces?

42. A loaded die has probabilities $\frac{1}{21}, \frac{2}{21}, \frac{3}{21}, \frac{4}{21}, \frac{5}{21}, \frac{6}{21}$ of showing 1, 2, 3, 4, 5, 6.

(a) What is the probability of throwing two 3's in succession?

(b) What is the probability of throwing a 4 the first time and not a 4 the second time?

(c) If two such dice are thrown and the sum of the numbers on the faces is greater than or equal to 10. what is the probability that both are 5's?

(d) How many times must we throw loaded die to have probability greater than $\frac{1}{2}$ of getting an ace?

(e) The loaded die is thrown twice. What is the probability that the number on the die is even the first time > 4 the second time?

43. Show the following results about rearrangement schemes for polynomial equations.

(a) If a polynomial equation $g(x) = x^m - f(x) = 0$ where $f(x)$ is a polynomial of degree less than m and for which $f(0) \neq 0$, is solved using a rearrangement scheme $x_{n+1} = (f(x_n))^{1/m}$, then is general the scheme will have only first order convergence.

(b) The same iteration scheme can give second or higher order convergence to the cubic equation $x^3 - ax^2 + 2abx - (b^3 + ab^2) = 0$, where $a \neq 0$ $b \neq 0$ in special cases.

44. Use the Runge-Kutta method of order four to solve for $y(0.1), y(0.2)$ and $y(0.3)$ given that $y' = ty + y^2$, $y(0) = 1$.