(Pages : 4) **K – 2377**

Reg. No. : ....

Name : ....

Third Semester B.Sc. Degree Examination, March 2021

First Degree Programme under CBCSS

Mathematics

Complementary Course for Chemistry & Polymer Chemistry

Mathematics – III

MM 1331.2 : VECTOR ANALYSIS AND THEORY OF EQUATIONS

(2015 – 2017 Admission)

Time : 3 Hours Max. Marks : 80

PART – A

Answer **all** questions. Each question carries **1** mark.

1. State fundamental theorem of algebra.

2. Form a rational cubic equation whose roots include 2 and 3 + *i*.

3. Solve the equation, $x^3 + x^2 - 2 = 0$ whose one of the root is 1.

4. Find the number of imaginary roots of $x^5 - x^4 - 4x - 1 = 0$.

5. Define curvature function of a curve.

6. Define the continuity of a vector function.

P.T.O.

7. Define curl $\overline{F}$.

8. Write down the condition that Mdx + Ndy + Pdx is exact.

9. If $\bar{r} + x\hat{i} + y\hat{j} + z\hat{k}$, evaluate $div\ \bar{r}$.

10. State Stoke's theorem.

(10 × 1 = 10 Marks)

PART – B

Answer **any eight** questions. Each question carries **2** marks.

11. Solve the equation $x^4 + 4x^3 + 5x^2 + 2x - 2 = 0$, of which one root is $-1+\sqrt{-1}$.

12. Given that the roots of the equation $4x^3 - 24x^2 + 23x + 18 = 0$ are in AP. Solve the equation completely.

13. Find the condition that the roots of $ax^3 + 3bx^2 + 3cx + d = 0$ may be in GP.

14. Solve the equation $x^3 - 7x^2 + 36 = 0$, given that the difference between two of the roots is 5.

15. Find the unit tangent vector of the helix $r(t) = (\cos t)\, i + (\sin t)\, j + tk$.

16. The position vector of a particle in space at time t is $r(t) = 3\cos ti + 3\sin tj + t^2k$. Find the velocity and acceleration vectors.

17. If $f(x, y, z) = 3x^2y - y^3z^2$, find grad f.

18. Find the flux of $F = (x - y)\, i + xj$ across the circle $x^2 + y^2 = 1$ in the xy-plane.

19. Find the work done by $F = zi + xj + yk$, over the curve $r(t) = (\sin t)\, i + (\cos t)\, j + tk,\ 0 \le t \le \pi$, in the direction of increasing t.

20. Evaluate $\int_C F \cdot dr$ where $F = (y - x^2)\, i + (z - y)\, j + (x - z^2)\, k$ along the curve C given by $r(t) = ti + t^2 j + t^3 k,\ 0 \le t \le 1$.

21. Show that curvature of a circle of radius a is $\frac{1}{a}$.

22. Determine the length of the curve $x = e^t \cos t,\ y = e^t \sin t,\ z = 0$ between $t = 0$ and $t = 1$.

(8 × 2 = 16 Marks)

PART – C

Answer **any six** questions. Each question carries 4 marks.

23. Two roots of the equation $x^4 - 6x^3 + 18x^2 - 30x - 25 = 0$ are of the form $\alpha + i\beta$ and $\beta + i\alpha$. Find all the roots of the equation.

24. Use Newton - Raphson method to obtain a root, correct to three decimal places of $x^3 - 5x + 3 = 0$.

25. Solve $x^4 + 2x^3 - 21x^2 - 22x + 40 = 0$, it being given that the sum of two of its roots is equal to the sum of the other two.

26. Describe the bisection method of finding the solution of a general function $f(x) = 0$.

27. Green's theorem calculate the area enclosed by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$.

28. Establish the relation Curl Curl $\bar{f} = \overline{\nabla} div\bar{f} - \nabla^2 \bar{f}$.

29. Show that, $F = 2xyzi + x^2zj + x^2yk$ is conservative and find a potential function for it.

30. A vector $\bar{r}$ is defined by $\bar{r} = xi + yj + zk$. If $|\bar{r}| = r$ then show that the vector $r^n\bar{r}$ is irrotational.

31. The velocity of a particle moving in space is $\frac{dr}{dt} = \cos t\ i + \sin t\ j + k$. Find the particle's position as a function of t if $r = 2i + k$ when $t = 0$.

(6 × 4 = 24 Marks)

PART – D

Answer **any two** questions. Each question carries **15** marks.

32. (a) Prove that every polynomial equation of degree n has n roots and no more.

(b) Solve the equation $2x^3 - x^2 - 22x - 24 = 0$, two of whose roots are in the ratio 3:4.

33. Find a positive root of the equation $xe^x = 1$, which lie between 0 and 1 and such that percentage error is less than 0.05%.

34. If $A = 2xyi + yz^2j + xzk$ and S is a rectangular parallelepiped bounded by $x = 0, y = 0, z = 0$; $x = 2, y = 1, z = 3$, verify divergence theorem.

35. Verify Stoke's theorem, when $F = (2x - y)\ i - yz^2j - y^2zk$, S is the upper half surface of the unit sphere $x^2 + y^2 + z^2 = 1$ and C is its boundary.

(2 × 15 = 30 Marks)