Reg. No. : ....

Name : ....

Third Semester B.Sc. Degree Examination, March 2022

First Degree Programme Under CBCSS

Mathematics

Complementary Course for Chemistry and Polymer Chemistry

MM 1331.2 : MATHEMATICS III – LINEAR ALGEBRA, PROBABILITY THEORY AND NUMERICAL METHODS

(2019 & 2020 Admission)

Time : 3 Hours

Max. Marks : 80

SECTION – I

All the first 10 questions are compulsory. Each carries **1** mark.

1. Define the transpose of a matrix.

2. Evaluate the determinant $\begin{vmatrix} 0 & 3 & -2 \\ -3 & 7 & -4 \\ 0 & 6 & 0 \end{vmatrix}$.

3. Write a symmetric equation of a straight line.

4. Define Kronecker δ.

5. What is the cross product of two vectors?

6. Give a definition of the concept 'Probability of an event E'.

P.T.O.

7. There are 10 questions and you are to do 8 of them. In how many ways can you choose them?

8. Write Baye's formula for conditional probability.

9. Write the expression for variance of a random variable x and explain the terms.

10. What is a transcendental equation?

<div align="right">(10 × 1 = 10 Marks)</div>

<div align="center">SECTION – II</div>

Answer any **eight** questions. Each carries **2** marks.

11. Evaluate the determinant $\begin{vmatrix} 1 & -5 & 2 \\ 7 & 3 & 4 \\ 2 & 1 & 5 \end{vmatrix}$.

12. Find a vector perpendicular to both the vectors $A = 2i + j - k$ and $B = i + 3j - 2k$.

13. Find the rank of the matrix $\begin{pmatrix} 2 & -3 & 5 & 3 \\ 4 & -1 & 1 & 1 \\ 3 & -2 & 3 & 4 \end{pmatrix}$.

14. Find the parametric equation of the line through $(1, -1, -5)$ and $(2, -3, -3)$.

15. Define linear operator.

16. Is square root a linear operator? Why?

17. A three digit number is selected at random. What is the probability that all three digits are the same?

18. Two dice are rolled. What is the probability that the sum is ≥ 10?

19. Define mutually exclusive events.

20. If $P(A) = 0.07755$, $P(AB) = 0.038$, find $P_A(B)$.

21. A card is selected from a shuffled deck. What is the probability that it is either a king or a club? That it is both a king and a club?

22. A club consists of 50 members. In how many ways can a president, vice president, secretary and treasurer be chosen?

23. Write an iteration scheme for finding the square root of X.

24. What is binary chopping?

25. Evaluate the integral $I = \int\limits_0^1 \dfrac{1}{1+x^2}\,dx$ using the Simpson's rule.

26. Write Newton-Raphson iteration formula.

$$(8 \times 2 = 16 \text{ Marks})$$

SECTION – III

Answer any **six** questions. Each carries **4** marks.

27. Evaluate the determinant $D = \begin{vmatrix} 0 & 1 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 & 0 \end{vmatrix}$.

28. Write and row reduce the augmented matrix for the equations :

$x - y + 4z = 5$
$2x - 3y + 8z = 4$
$x - 2y + 4z = 9.$

29. Using Cramer's rule solve the set of equations :

$2x + 3y = 3$

$x - 2y = 5$

30. Find the equation of a line through $(1, 0, -2)$ and perpendicular to the plane $3x - 4y + z + 6 = 0$.

31. Find the distance between the lines $r = i - 2j + (i - k)t$ and $r = 2j - k + (j - i)t$.

32. Which is the most probable sum in a toss of two dice? What is its probability?

33. Two students are working separately on the same problem. If the first student has probability $\frac{1}{2}$ of solving it and the second student has probability $\frac{3}{4}$ of solving it, what is the probability that atleast one of them solves it?

34. Find the coefficient of x^8 in the binomial expansion of $(1 + x)^{15}$.

35. Using Baye's formula find the probability of all heads in three tosses of a coin if you know that atleast one is a head?

36. Find an explicit formula that will generate a random number y distributed on $(-\infty, \infty)$ according to the Cauchy distribution $f(y)dy = \left(\frac{a}{\pi}\right)\frac{dy}{a^2 + y^2}$, given a random number ξ uniformly distributed on $(0, 1)$.

37. Evaluate $I = \int\limits_{0}^{2}(x^2 - 3x + 4)dx$ using trapezium rule with $h = 0.5$.

38. Find the numerical solution of the equation $\frac{dy}{dx} = 2y^{\frac{3}{2}}$, $y(0) = 1$, for $x = 0.1$ to 0.5 in steps of 0.1.

(6 × 4 = 24 Marks)

SECTION – IV

Answer any **two** questions. Each carries **15** marks.

39. Find the rotation matrix C if the quadratic surface $x^2 + 6xy - 2y^2 - 2yz + z^2 = 24$ is rotated to principal axis.

40. Find the eigen values and eigen vectors of the matrix $M = \begin{pmatrix} 1 & -4 & 2 \\ -4 & 1 & -2 \\ 2 & -2 & -2 \end{pmatrix}$.

41. If two dice are rolled :

    (a) What is the probability that the sum of the numbers on the dice will be 5? (3)

    (b) What is the probability that the sum on the dice is divisible by 5? (3)

    (c) What is the most probable sum in a toss of two dice? What is its probability? (3)

    (d) What is the probability that the sum on the dice is greater than or equal to 9? (3)

    (e) If the sum is odd, what is the probability that it is equal to 7? (3)

42. Derive the Poison density function $P_n = \dfrac{\mu^n}{n!} e^{-\mu}$.

43. Explain :

    (a) Stratified sampling method. (5)

    (b) Control variates method. (5)

    (c) Hit or miss method. (5)

44. Solve the simultaneous equations :

$$x_1 + 6x_2 - 4x_3 = 8$$
$$3x_1 - 20x_2 + x_3 = 12$$
$$-x_1 + 3x_2 + 5x_3 = 3$$

using Gaussian elimination.

(2 × 15 = 30 Marks)

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