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N – 3966

Reg. No. :

Name :

First Semester B.Sc. Degree Examination, June 2022

First Degree Programme under CBCSS

Mathematics

Core Course

MM 1141 : METHODS OF MATHEMATICS

(2014–2017 Admission)

Time : 3 Hours

Max. Marks : 80

SECTION – I

Answer the first ten questions are compulsory. These questions carry 1 mark each.

1. When we say that 2 integers a and b are congruent modulo m ?
2. Give example for co-prime numbers.
3. Define Fermat numbers.
4. For what values of x is $y = x^2 - 6x + 8$ is zero.
5. State the graph of $y = \sqrt{x - 3}$.
6. State intermediate value theorem.
7. Find $\lim_{x \rightarrow 2} \frac{x^2 - 4}{x - 2}$

P.T.O.

8. Find $\frac{d}{dx} (x^{-7/8})$

9. Sketch the parabola $x^2 = 4y$.

10. Find the vertices of the ellipse $16x^2 + 25y^2 = 400$

(10 × 1 = 10 Marks)

SECTION – II

Answer any **eight** questions. These question carries **2** marks.

11. Find $12^{39} \pmod{13}$.

12. If e/a and e/b , then prove that $e/(a, b)$

13. Prove that if $P(n)$ can be proved for every $n \geq n_0$ by ordinary induction, then it can be proved by complete induction.

14. Prove that every composite number is divisible by some prime number.

15. Find the domain and range of $f(x) = \frac{x+1}{x-1}$.

16. Find the equation of tangent line to $y = x^2 + 1$ at $x = 2$.

17. Find the horizontal asymptote of the graph of $f(x) = \frac{5x^2 + 8x - 3}{3x^2 + 2}$.

18. Is $[x]$, the greatest integer function is continuous everywhere? Justify.

19. For what value of x is there a hole or a gap in the graph of $y = \frac{x^2 - 9}{x^2 - 5x + 6}$.

20. Find $\frac{dw}{dt}$ if $w = \tan x$ and $x = 4t^3 + t$.

21. Find the equation of an ellipse with foci $(0, \pm 2)$ and major axis with end points $(0, \pm 4)$.
22. Find the new co-ordinates of the point $(2, 4)$ if the co-ordinate axes are rotated through an angle 30° .

(8 × 2 = 16 Marks)

SECTION – III

Answer any **six** questions. These question carries **4** marks each.

23. Prove that there are infinitely many primes.
24. State and prove division theorem.
25. Prove that the number $2n^3 - 3n^2 + n + 31 \geq 0$ for every $n \geq -2$.
26. If $f(x) = \sqrt{x}$ and $g(x) = \sqrt{1-x}$, then find $f+g$, $f-g$, $f \circ g$ and f/g .
27. Find the graph of the parametric equation $x = \cos t$ and $y = \sin t$, $0 \leq t \leq 2\pi$.
28. Prove that $\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$.
29. Find $\frac{d^2y}{dx^2}$ at $x = 0$ if $y = \frac{\sin x}{1 + \cos x}$.
30. Describe the graph of the equation $y^2 - 8x - 6y - 23 = 0$.
31. Find the directrix of the parabola $r = \frac{25}{10 + 10 \cos \theta}$.

(6 × 4 = 24 Marks)

SECTION – IV

Answer any **two** questions. These questions carries **15** marks.

32. (a) State and prove the fundamental theorem of arithmetic.
- (b) Prove that the relation 'Congruence Modulo' is an equivalence relation.

33. (a) Show that $f(x) = \begin{cases} x \sin \frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ is continuous, but not differentiable at $x = 0$.
- (b) Let $y = x^2 + 1$. Find the average rate of change of y with respect to x over the interval $[3, 5]$. Also find the instantaneous rate of change at $x = -4$.
34. (a) Assume that the oil spilled from a ruptured tanker spreads in a circular pattern whose radius increases at a constant rate of 2 ft/s . How fast is the area of the spill increasing when the radius of the spill is 60 ft ?
- (b) Consider a ball which is thrown vertically upwards so that the height (in feet) of the ball above the ground in t seconds after its release is modelled by the function. $s(t) = 16t^2 + 29t + 6$, $0 \leq t \leq 2$. Find the velocity function $v(t)$ for $0 < t < 2$. What is the velocity of the ball just before impacting the ground at time $t = 2 \text{ s}$?
35. (a) Sketch the graph of $r = \frac{6}{2 + \cos \theta}$ in polar co-ordinates.
- (b) Find the equation of the hyperbola with vertices $(0, \pm 8)$ and asymptotes $y = \pm \frac{4}{3}x$. Also sketch the hyperbola.

(2 × 15 = 30 Marks)