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Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, April 2022

First Degree Programme under CBCSS

Mathematics

Core Course

MM 1641 : REAL ANALYSIS – II

(2014 & 2017 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the first **10** questions are compulsory. Each carries **1** mark.

1. State sequential criterion for continuity.

2. Give an example for a function f such that f is not continuous at any point of its domain.

3. Find $g'(x)$ if $g(x)=\sqrt{5-2x+x^2}$.

4. Evaluate $\lim_{x\to 0}\frac{e^x-1}{x}$.

5. Find $f^{(n)}(x)$ for $n\in\mathbb{N}$, if $f(x)=\cos ax, x\in\mathbb{R},\ a\neq 0$.

6. Give an example for a non- Riemann integrable function.

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7. If $F(x)=\int_0^{x^2}\left(1+t^3\right)^{-1}dt$ then find $F'(x)$.

8. State substitution theorem.

9. Define a null set.

10. *If* $F(x)=1$ for $x=\frac{1}{5},\frac{2}{5},\frac{3}{5},\frac{4}{5}$ and $F(x)=0$ *elsewhere* on [0,1] evaluate $\int_0^1 F$.

SECTION – II

Answer any **eight** questions. **Each** question carries **2** marks.

11. Let $I \subseteq \mathbb{R}$ be an interval and $f: I \to \mathbb{R}$ be increasing on I. Suppose $c \in I$ is not an end point of I. Then prove that $\lim_{x\to c^-} f = \sup\{f(x): x \in I, x < c\}$.

12. If $I = [a, b]$ is a closed and bounded interval and $f: I \to \mathbb{R}$ is continuous on I, prove that f is bounded on I.

13. Let I be an interval and $f: I \to \mathbb{R}$ is continuous on I. Then show that $f(I)$ is an interval.

14. Show that both $f(x)=x$ and $g(x)=x-1$ are increasing on [0,1], but their product fg is not increasing on [0,1].

15. Find $f'(0)$, if $f(x)=\begin{cases} x^2\sin\left(\frac{1}{x}\right), & \text{for } x \neq 0 \\ 0 & \text{for } x = 0 \end{cases}$.

16. Show that every constant function on [a, b] is in R[a, b].

17. If $f,g \in R[a,b]$ show that $f+g \in R[a,b]$.

18. Show that every countable set is a null set.

19. Use the mean value theorem theorem to find approximate value of $\sqrt{105}$.

20. If $0 < a < 1$and $a > 0$, $b > 0$ show that $a^{\alpha} b^{1-\alpha} \leq \alpha a + (1-\alpha) b$ where equality holds if and only if a = b.

21. Evaluate $\lim\limits_{x \to 0}+ \dfrac{ln \sin x}{ln x}$.

22. State Lebesgue's Integrability Criterion. Use the result to show that Thomae's function is Riemann Integrable.

SECTION – III

Answer any **six** questions. **Each** question carries **4** marks.

23. State and prove continuous inverse theorem.

24. If $f : I \to \mathbb{R}$ and $c \in 1$, prove that f is differentiable at c if q and only if, there exists a function φ on I that is continuous at c and satisfies $f(x) - f(c) = \varphi(x)(x - c)$. In that case show that $\varphi(c) = f'(c)$. Find the function $\varphi(x)$ for $f(x) = x^n$, $x \in \mathbb{R}$.

25. State and prove fundamental Theorem form 1. Use fundamental theorem to evaluate $\int_{-10}^{10} \operatorname{sgn}(x) dx$.

26. State and Prove Bolzano's Intermediate Value Theorem.

27. State and prove chain rule for differentiation.

28. Let c be an interior point of the interval I at which $f : I \to \mathbb{R}$ has a relative extremum. If the derivative of f at c exists show that $f(c) = 0$.

29. Let $g : \mathbb{R} \to \mathbb{R}$ be defined by $g(x) = x + 2x^2 \sin\left(\dfrac{1}{x}\right)$, for $x \neq 0$ and $g(0) = 0$. Show that g is not monotonic in any neighborhood of 0.

30. Prove that $1 - \dfrac{1}{2}x^2 \leq \cos x$ for all $x \in \mathbb{R}$.

31. If $f \in \mathfrak{R}$ [a,b] show that the value of the integral is uniquely determined.

SECTION – IV

Answer any **two** questions. Each question carries **15** marks.

32. (a) If $I = [a,b], f : I \to \mathbb{R}$ is continuous on I and if $f(a) < 0 < f(b)$, then prove that there exists a number $c \in (a,b)$ such that $f'(c) = 0$.

    (b) Find the roots of $f(x) = xe^x - 2 = 0$ with an error less than 10^{-2}.

33. (a) State and prove Squeeze Theorem.

    (b) If $f : [a,b] \to \mathbb{R}$ is monotone on $[a, b]$, then prove that $f \in \mathfrak{R}[a, b]$.

34. (a) Explain briefly the tagged partition, norm of a partition and the Riemann sum. Define a Riemann Intergrable function on a closed bounded interval in $\mathbb{R}$.

    (b) Prove that $h(x) = x, x \in [0,1]$ is Riemann Integrable. Evaluates $\int_0^1 h$.

35. (a) State and prove Taylor's Theorem.

    (b) Use Taylor's Theorem with $n = 2$ to approximate $\sqrt[3]{1+x}$, $x > -1$.