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(Pages : 4) **N – 1288**

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, April 2022

First Degree Programme Under CBCSS

Mathematics

Core Course X

MM 1642 – COMPLEX ANALYSIS II

(2018 & 2019 Admission)

Time : 3 Hours Max. Marks : 80

PART – A

Answer **all** questions. **Each** question carries **1** mark.

1. When does a complex series diverge?
2. Define Uniform Convergence of a series of functions.
3. Explain the term: Cauchy sequence.
4. Define a zero of order m for a function f.
5. What is the formula for

   $\operatorname{Res}(f; z_0)$, if $f(z) = \frac{p(z)}{Q(z)}$, where $P(z)$ and $Q(z)$ are analytic at z_0

   and Q has a simple pole at z_0, while $P(z_0) \neq 0$.
6. Define Improper integral over $(-\infty, 0)$ of a continuous function $f(x)$.

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7. Define p.v. $\int_{-\infty}^{\infty} f(x)dx$.

8. What do you mean by global property of a mapping?

9. Are analytic functions that are non constant in domains being open mappings?

10. Define the mapping : Rotation.

(10 × 1 = 10 Marks)

PART – B

Answer **any eight** questions. **Each** question carries **2** marks.

11. Prove that $1+c+c^2+c^3+\cdots=\frac{1}{1-c}$ for $|c|<1$.

12. Find the Maclaurin series for cos *z*.

13. How can we obtain the Taylor series of *fg*, if *f* and *g* are two analytic functions?

14. State a necessary and sufficient condition for the convergence of a sequence of complex numbers.

15. Explain the term: Laurent Series

16. State the necessary and sufficient condition for an analytic function to have a pole of order m at z_0.

17. State the Picard's Theorem.

18. Define of a residue of a function with an example.

19. Find the residue at $z=0$ of $f(z)=ze^{\frac{3}{z}}$ using Laurent series.

20. Find the residue at $z=-3i$ of $f(z)=\frac{z+1}{z^2+9}$.

21. Evaluate $p.v. \int_{-\infty}^{\infty} x^3 dx$.

22. Give the statement of Jordan's Lemma.

23. Is $f(z)=z^2$ being locally one to one on a neighbourhood of 0? Justify your answer.

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24. Write the statement of Riemann Mapping Theorem.

25. Define magnification and show that it rescales distance.

26. State the symmetry principle of Mobius transformations.

(8 × 2 = 16 Marks)

PART – C

Answer **any six** questions. **Each** question carries **4** marks.

27. State the Ratio test and show that $\sum_{j=1}^{\infty}\frac{4^j}{j!}$ converges.

28. If $f(z)$ is analytic at z_0, how can we find the Taylor series of $f'(z)$. Using this result, find the Taylor series of cos z from that of $z=\sum_{j=1}^{\infty}\frac{z^{2j-1}}{(2j-1)!}$.

29. Prove that the uniform limit of a sequence of analytic functions defined on a simply connected domain is also analytic.

30. Classify the zeros and singularities of $\frac{\tan z}{z}$.

31. What do you mean by extended complex plane? Classify the behaviour at ∞ of $f(z)=\frac{iz+1}{z-1}$.

32. Find the residues at each singularity of $f(z)$ = *cosec z*.

33. State and prove Cauchy Residue Theorem.

34. Prove that if $|f(t)|\leq M(t)$ on $a\leq t\leq b$, then $\left|\int_a^b f(t)dt\right|\leq\int_a^b M(t)dt$ where $f(t)$ and $M(t)$ are continuous function defined on $[a, b]$, with f complex and M real valued.

35. Find the integral of $\frac{\sin x}{x}$ over $(0,\infty)$.

36. Prove that if $f(z)$ is analytic at z_0, then there is an open disk D centred at z_0 such that f is one to one on D.

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37. Define linear transformation and find the linear transformation that maps the circle $|z-1|=1$ onto the circle $\left|w-\frac{3i}{2}\right|=2$.

38. Show that the composition of two Mobius transforms is another Mobius Transform.

(6 × 4 = 24 Marks)

PART – D

Answer **any two** questions. **Each** question carries **15** marks.

39. Define Taylor Series of $f(z)$ around z_0. Prove that if $f(z)$ is analytic in the $|z-z_0|<R$ there exists a Taylor Series which converges to $f(z)$ for all z in this disk.

40. Find The Laurent series expansion of $f(z) = \frac{1}{(z-1)(z-2)}$ in

    (a) The region $|Z|<1$

    (b) The region $1<|Z|<2$

    (c) The region $|Z|>2$.

41. Define different singularities of a complex function with examples. Verify the examples with definition.

42. (a) Prove that, if $f(z)$ has a pole of order m at z_0, then

$$\mathrm{Re}\,s(f:z_0)=\lim_{z\to z_0}\frac{1}{(m-1)!}\frac{d^{m-1}}{dz^{m-1}}\left[(z-z_0)^m f(z)\right]$$

    (b) Using the result find the residue at $z=0$ of $f(z)=\frac{\cos z}{z^2(z-\pi)^3}$.

43. Evaluate $\int_0^x \frac{1}{2-\cos\theta}d\theta$.

44. (a) Prove that if $f(z)$ is analytic at z_0 and $f'(z_0)\neq 0$ then $f(z)$ is conformal at z_0.

    (b) Find all Mobius transformations that map unit disks into unit disks.

(2 × 15 = 30 Marks)
