(Pages : 4) **N – 1298**

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, April 2022.

First Degree Programme under CBCSS

Mathematics

MM 1661.1 – GRAPH THEORY

(2018 & 2019 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – I

Answer **all** the questions. Each question carries **1** mark.

1. Define a graph.
2. Draw a complete graph on 6 vertices.
3. Define a bipartite graph.
4. Define adjacency matrix of a graph.
5. State Cayley Theorem.
6. Define cut vertex of a graph.
7. Define an Eulerian graph.
8. Give an example for a polyhedral graph.
9. Draw a non- Hamiltonian graph containing a Hamiltonian path.
10. State Kuratowski's theorem.

(10 × 1 = 10 Marks)

P.T.O.

SECTION – II

Answer **any eight** questions. Each question carries **2** marks

11. Give 2 drawings of $K_{3,3}$ which are isomorphic.

12. Define a regular graph. Give one example.

13. Define an edge deleted subgraph. Give an example.

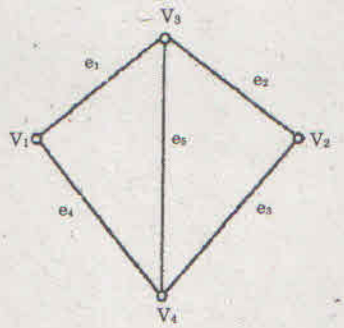
14. Define a tree. Draw all non-isomorphic trees with 5 vertices.

15. Write $\omega(G)$ for a connected and disconnected graph.

16. Let G be a connected graph. If every edge of G is a bridge, then prove that G is a tree.

17. Define a bridge. Give an example.

18. Write the incidence matrix of the following graph



19. Explain Konigsberg Bridge Problem.

20. Explain Chinese Postman Problem in graph theoretical terms.

21. Define closure of a graph. Find the closure of C_4.

22. Let G_1 and G_2 be 2 plane graphs which are both redrawings of the same planar graph. Then prove that G_1 and G_2 have the same number of faces.

23. Is K_4 Hamiltonian. Justify your answer

24. Draw a connected plane graph and verify Euler's Formula for the graph.

25. Draw a graph G satisfy the relation $k(G) = k(G * e)$.

26. Which are the only polyhedral graphs which are regular,

($8 \times 2 = 16$ Marks)

SECTION – III

Answer **any six** questions. Each question carries **4** marks

27. State and prove first theorem on Graph theory.

28. Define (a) eccentricity of a vertex (b) radius of a graph (c) diameter of a graph (d) Find the radius and diameter of the Peterson Graph

29. Prove that every u-v walk contains a u-v path for any 2 vertices u and v of a graph G.

30. Let G be a graph with n vertices $v_1, v_2, \ldots v_n$ and let A denotes the adjacency matrix of G with respect to the listing of vertices. Let k be any positive integer and let A^k denote the matrix multiplication of k copies of A. Prove that the $(i,j)^{th}$ entry of A^k is the number of different $v_i - v_j$ walks in G of length k.

31. Let G be a graph with $n \geq 2$ vertices. Then prove that G has at least 2 vertices which are not cut vertices.

32. If a graph G is connected, then prove that it has a spanning tree.

33. Prove that a simple graph is Hamiltonian if and only if its closure is Hamiltonian.

34. Prove that K_5 is non planar.

35. Let G be a connected simple planar graph with $n \geq 3$ vertices and e edges, Prove that $e \leq 3n - 6$.

36. Let P be a convex polyhedron and G be its corresponding polyhedral graph. Prove that P and so the graph G has at least one face bounded by a cycle of length n for either n = 3,4 or 5.

37. Explain Travelling salesman Problem

38. Prove that a connected graph has an Euler trail if and only if it has at most two odd vertices.

($6 \times 4 = 24$ Marks)

SECTION – IV

Answer **any two** questions. Each question carries **15** marks

39. (a) Prove that a tree with n vertices has precisely $n-1$ edges.

    (b) Prove that an edge e of a graph G is a bridge if and only if e is not any part of any cycle in G.

40. State and Prove Whitney's Theorem.

41. Let G be a non-empty connected graph with at least 2 vertices. Prove that G is bipartite if and only if G contains no odd cycle.

42. Prove that a connected graph is Euler if and only if degree of every vertex is even.

43. State and Prove Dirac Theorem.

44. (a) Define (i) Subdivision of a graph (ii) Contraction on an edge, with examples.

    (b) Let G be a simple 3-connected graph with at least 5 vertices. Then prove that G has a contractible edge

(2 × 15 = 30 Marks)