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Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, April 2022

First Degree Programme Under CBCSS

Mathematics

Core Course IX

MM 1641 – REAL ANALYSIS – II

(2018 & 2019 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – A

All the first ten questions are compulsory. They carry 1 mark each.

1. Evaluate $\lim_{x \to \infty} \left(1 + \frac{1}{n}\right)^n$.

2. State true or false: A function that is continuous on a compact set K is uniformly continuous on K.

3. Define a bounded function.

4. Determine the points of discontinuity of the Dirichlet's function.

5. State Rolle's theorem.

6. State intermediate value property.

7. When do you say that a function is differentiable on an interval?

P.T.O.

8. State true or false: If f is differentiable in $[a,b]$ and $f'(x) = 0$ for all $x \in (a,b)$, then f is continuous.

9. Define lower integral of a function f.

10. Compute $\int_0^3 [x]\, dx$, where $[x]$ denotes the greatest integer function.

SECTION – B

Answer **any eight** questions. Each question carries **2** marks.

11. Show that the limit $\lim_{x \to 2} \frac{|x-2|}{x-2}$ doesn't exist.

12. Let f and g be real valued functions then prove that

$$\lim_{x \to c} \{f(x) + g(x)\} = \lim_{x \to c} f(x) + \lim_{x \to c} g(x).$$

13. Show that $|x|$ is continuous everywhere.

14. Let $[x]$ denote the largest integer containing in x and $(x) = x - [x]$ denote the fractional part of x. What discontinuity do the function (x) has?

15. If, f, g be two functions continuous at a point c then the function f-g is also continuous at c.

16. Show that the function $f(x) = x^2$ is uniformly continuous on [–1, 1].

17. Suppose that $\{x_n\}$ is a Cauchy sequence in R. Prove that $f(x_n)$ is a Cauchy sequence where f is a uniformly continuous function.

18. State Squeez theorem.

19. Give an example to show that continuous function need not be differentiable.

20. If f is differentiable in $(a,b)$and $f'(x) \geq 0$ for all $x \in (a,b)$, show that f is monotonically increasing.

21. Suppose f and g are defined on [a, b] and are differential at a point $x \in [a,b]$. Prove that $f + g$ is differentiable.

22. Show that $\int_{\underline{a}}^{b} f dx \le \int_{a}^{\overline{b}} f dx$.

23. Show that the function $f(x)$ defined on **R** by $f(x) = \begin{cases} x \text{ if } x \text{ is irrational} \\ -x \text{ if } x \text{ is rational} \end{cases}$ is continuous only at $x = 0$.

24. If P_1 and P_2 are any two partitions of $[a, b]$,then $L(f, P_1) \le U(f, P_2)$.

25. If a function *f* is integrable on $[a, b]$ and $m \le f(x) \le M$ for $x \in [a, b]$, prove that $m(b - a) \le \int_a^b f \le M(b - a)$.

26. Show that if *f* and *g* are bounded and integrable on $[a, b]$ such that $f \le g$ then $\int_a^b f\ dx \le \int_a^b g\ dx$.

SECTION – C

Answer **any six** questions. Each question carries **4** marks.

27. Show that $\lim_{x \to 0} \sin\left(\frac{1}{x}\right)$ does not exist.

28. Assume *f* and g are defined on all of *R* and that $\lim_{x \to p} f(x) = q$ and $\lim_{x \to q} g(x) = r$. Give an example to show that it may not be true that $\lim_{x \to p} g\ (f(x)) = r$.

29. Prove that a function which is continuous on a closed interval is uniformly continuous on that interval.

30. If *f* is a real differentiable function on $[a, b]$ and suppose $f'(a) < \lambda < f'(b)$ then prove that there is a point $x \in (a, b)$ such that $f'(x) = \lambda$.

31. State and prove extreme value theorem.

32. Prove that if *f* is differentiable on an open interval in (*a*, *b*) and *f* attains a maximum value at some point *c* in (*a*, *b*), then $f'(c) = 0$.

33. Show that $\int_{3}^{4} f dx = \frac{41}{2}$ where $f(x) = 5x + 3$.

34. If $g : A \to R$ is differentiable on an interval A and satisfies $g'(x) = 0$ for all $x \in A$, then prove that $g(x) = k$ for some constant $k \in R$.

35. Prove that a continuous function in a closed interval is integrable in that interval.

36. Prove that if f is monotonic in $[a, b]$ then f is integrable in $[a, b]$.

37. If f is bounded and integrable in [a, b], prove that there exists a number μ lying between a and b such that $\int_{a}^{b} f(x)\, dx = \mu(b - a)$.

38. Assume f is integrable function on the interval $[a, b]$, then show that $|f|$ is also integrable and $\left|\int_{a}^{b} f\right| \le \int_{a}^{b} |f|$.

SECTION – D

Answer **any two** questions. Each question carries **15** marks.

39. State and prove intermediate value theorem. Is the converse true? Justify.

40. Define Lipschitz functions. Show that every Lipschitz function is uniformly continuous. Is the converse statement true? Justify.

41. State and prove chain rule for differentiation.

42. (a) State and prove Mean value theorem.

    (b) Prove that if f is continuous in [a, b], then f is integrable in [a, b].

43. If $f : [a, b] \to R$ is bounded, and f is integrable on $[c, b]$ for all $c \in (a, b)$, then prove that f is integrable on $[a, b]$.

44. State and prove fundamental theorem of calculus.
