N – 1297

Reg. No. : .....

Name : .....

Sixth Semester B.Sc. Degree Examination, April 2022

First Degree Programme Under CBCSS

Mathematics

Elective Course

MM 1661.1 GRAPH THEORY

(2014 & 2017 Admission)

Time : 3 Hours

Max. Marks : 80

SECTION – I

All the first **10** questions are compulsory. They carry **1** mark each

1. Define regular graph.
2. What is a pendant vertex?
3. Give an example of a self complementary graph.
4. Define inward demi-degree.
5. Define walk in a graph.
6. If G is an Euler graph then all vertices of G are of ———— degree.
7. Is the following graph an Euler digraph?

8. Define diameter of a graph.

9. In any tree there are at least ———— pendant vertices.

10. Define spanning tree of a graph *G*.

SECTION – II

Answer any **eight** questions. These questions carry **2** marks each.

11. Prove that in any graph G, the number of odd vertices is always even.

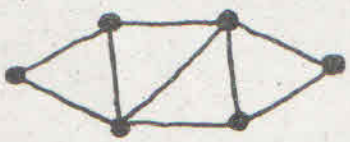
12. Draw any two graphs having equal number of vertices, edges and same degree sequence, but are non-isomorphic.

13. Write the incidence matrix of the following graph.

14. Show that the maximum number of edges in a complete bipartitle graph on n vertices is $\frac{n^2}{4}$.

15. Prove that, if a graph G has exactly two vertices of odd degree then there must be a path joining these two vertices.

16. Is the following graph Eular graph. Explain.



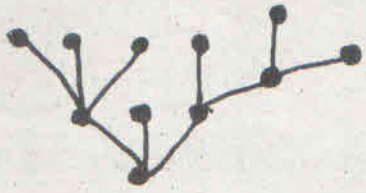
17. Explain Teleprinter's problem.

18. Draw all non-isomorphic graphs on 4 vertices.

19. Prove that, if in a graph G there is one and only one path between every pair of vertices, then G is a tree.

20. Prove that in any tree T there are at least two vertices of degree one.

21. Find the center of the following tree.



22. Prove that the distance between vertices of a connected graph is a metric.

SECTION – III

Answer any **six** questions. These questions carry **4** marks each.

23. Define complement of a graph, graph isomorphism and self-complementary graph. Give an example of a self-complementary graph.

24. Define spanning subgraph and induced subgraph. Is K_3 a spanning subgraph of K_4? Is it an induced subgraph? Explain.

25. Define adjacency matrix. Draw the graph with the adjacency matrix.

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix}.$$

26. Prove that G is an Euler graph if and only if ever vertex is of even degree.

27. Prove that a simple disconnected graph has at most $\frac{(n-1)(n-2)}{2}$ edges.

28. Prove that, given any spanning tree T of a connected graph G with n vertices and e edges has $n=1$ branches and $e-n+1$ chords.

29. Prove that $K_{3,3}$ is non-planar.

30. Draw planar representations of K_5 minus an edge and $K_{2,3}$.

31. Prove that a graph can be embedded in the surface of a sphere if and only if it can be embedded in a plane.

SECTION – IV

Answer **any two** questions. These questions carry **15** marks each.

32. (a) Show that the maximum number of edges in a simple graph on n vertices is $\frac{n(n-1)}{2}$.

(b) Show that an infinite graph with a finite number of vertices will have at least one pair of vertices joined by an infinite number of parallel edges.

(c) Sketch 3 non-isomorphic trees on 5 vertices.

33. (a) In a connected graph G with exactly 2k odd vertices, prove that there exist k edge disjoint subgraphs such that they together contain all edges of G and that each is a unicursal graph.

(b) Explain Königsberg Bridge Problem.

34. (a) Prove that there is one and only one path between every pair of vertices in a tree T.

(b) Prove that a graph with n vertices, n – 1 edges and no circutes is connected.

(c) Prove that in any tree T there are at least two vertices of degree one.

35. What are Kuratowski's graphs? Prove that they are non-planar.
