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N – 1294

Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, April 2022

First Degree Programme under CBCSS

Mathematics

Core Course XII

MM 1644 : LINEAR ALGEBRA

(2018 and 2019 Admission)

Time : 3 Hours

Max. Marks : 80

SECTION – A

Answer **all** questions. Each carries **1** mark.

1. Write the following system of equations in column form.

$$2x + 3y = 10$$
$$4x - 5y = 11$$

2. Describe the intersection of the three planes $u + v + w + z = 6$, $u + w + z = 4$ and $u + w = 2$, all in four dimensional space.

3. Define a skew symmetric matrix.

4. Define column space of a matrix.

5. If $v_1, v_2, ... v_n$ are linearly independent, the space they span has dimension __________

6. Write the rotation matrix that turns all vectors in the xy plane through 90°.

P.T.O.

7. If a 4 by 4 matrix A has $\det A = \frac{1}{2}$, find $\det(2A)$.

8. State whether true or false : If $\det A = 0$, then at least one of the cofactors must be zero.

9. The eigen values of a projection matrix are ________.

10. State principal axis theorem.

SECTION – B

Answer **any eight** questions. Each carries **2** marks.

11. Draw the two pictures in two planes for the equations $x - 2y = 0, x + y = 6$.

12. Find the inner product $\begin{bmatrix} 1 & -2 & 7 \end{bmatrix} \begin{bmatrix} 1 \\ -2 \\ 7 \end{bmatrix}$.

13. Write a 2 by 2 matrix A such that $A^2 = -I$.

14. Prove that $(AB)^{-1} = B^{-1}A^{-1}$.

15. Describe the column space of the matrix $A = \begin{bmatrix} 0 & 0 & 3 \\ 1 & 2 & 3 \end{bmatrix}$.

16. Determine whether the vectors (1, 0, 0), (–1, 2, 1) and (2, 1, 1) are linearly independent.

17. Describe the subspace of R^3 spanned by the three vectors (0, 1, 1), (1, 1, 0) and (0, 0, 0).

18. Prove that a linear transformation leaves the zero vector fixed.

19. Find the determinant of $A = \begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix} \begin{bmatrix} 2 & -1 & 2 \end{bmatrix}$.

20. Using the big formula, compute the determinant of $A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 3 & 2 & 1 \end{bmatrix}$ from six terms. Are rows independent.

21. Find the area of the parallelogram with edges $v = (3, 2)$ and $w = (1, 4)$.

22. Suppose the permutation p takes (1, 2, 3, 4, 5) to (5, 4, 1, 2, 3). What does p^2 do to (1, 2, 3, 4, 5)?

23. Suppose A and B have the same eigen values $\lambda_1, \ldots. \lambda_m$ with the same independent eigen vectors $x_1, \ldots.. x_m$. Show that $A = B$.

24. Describe all matrices S that diagonalize the matrix $A = \begin{bmatrix} 4 & 0 \\ 1 & 2 \end{bmatrix}$.

25. Prove that every third Fibonacci number in 0, 1, 1, 2, 3,... is even.

26. Prove that if $A = A^H$, then every eigen value is real.

SECTION – C

Answer **any six** questions. Each carries **4** marks.

27. Find a coefficient 'b' that makes the following system singular. Then choose a right hand side 'g' that makes it solvable. Find two solutions in that singular case.

28. The parabola $y = a + bx + cx^2$ goes through the points $(x, y) = (1, 4), (2, 8)$ and $(3, 14)$. Find and solve a matrix equation for the unknowns (a, b, c).

29. Solve $Lc = b$ to find c. Then solve $Ux = c$ to find x.

$$L = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}, U = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \text{ and } b = \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix}.$$

30. Reduce to echelon form and find the rank $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \\ 5 & 5 & 8 \end{bmatrix}$.

31. What are the special solutions to $Rx = 0$ and $R^T y = 0$ for the given R.

$$R = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}.$$

32. Find the range and kernel of T.

$$T(v_1, v_2, v_3) = (v_1 + v_2, v_2 + v_3, v_1 + v_3)$$

33. By applying row operations, produce an upper triangular matrix u and compute

$$\det \begin{bmatrix} 1 & 2 & 3 & 0 \\ 2 & 6 & 6 & 1 \\ -1 & 0 & 0 & 3 \\ 0 & 2 & 0 & 7 \end{bmatrix}.$$

34. Find x, y and z by Cramer's rule.

$$x - 3y + z = 2$$
$$3x + y + z = 6$$
$$5x + y + 3z = 3$$

35. If every row of a matrix A adds to 1, prove that $\det(A - I) = 0$. Show by an example that this doesn't imply $\det A = 1$.

36. Find the eigen values and eigen vectors of $A=\begin{bmatrix}3 & 4 & 2\\0 & 1 & 2\\0 & 0 & 0\end{bmatrix}$.

37. Prove that Diagonalizable matrices share the same eigen vector matrix S if and only if $AB = BA$.

38. Write out the matrix A^H and compute $C = A^H A$ if $A=\begin{bmatrix}1 & i & 0\\i & 0 & 1\end{bmatrix}$.

SECTION – D

Answer **any two** questions. Each carries **15** marks.

39. (a) Reduce the system to upper triangular form and solve by back substitution for z, y, x.

$$2x-3y=3$$
$$4x-5y+z=7$$
$$2x-y+3z=5$$

(b) Use Gauss Jordan method to find A^{-1}

$$A=\begin{bmatrix}2 & 1 & 1\\4 & -6 & 0\\-2 & 7 & 2\end{bmatrix}$$

40. (a) What are L and D for the matrix A. What is U in $A = Lu$ and what is the new U in $A = LDu$?

$$A=\begin{bmatrix}2 & 4 & 8\\0 & 3 & 9\\0 & 0 & 7\end{bmatrix}$$

(b) Compute LDL^T factorisation of $\begin{bmatrix}1 & 2\\2 & 8\end{bmatrix}$.

41. (a) Find the value of c that makes it possible to solve $Ax = b$ and solve it :

$$u + v + 2w = 2$$
$$2u + 3v - w = 5$$
$$3u + 4v + w = c$$

(b) Find the dimensions of the column space and row space of $A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 3 & 1 \\ 3 & 1 & -1 \end{bmatrix}$.

42. (a) Find the dimensions and a basis for the four fundamental subspaces for $A = \begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 2 & 0 & 1 \end{bmatrix}$.

(b) Check whether the mapping $T : R^2 \to R^2$ given by

$T(v_1, v_2) = (v_1 + v_2, v_1 - v_2, v_2)$ is linear.

43. Find the determinant and compute the cofactor matrix C. Verify that $AC^T = (\det A)I$. What is A^{-1}?

44. (a) Test the Cayley Hamilton theorem on $A = \begin{bmatrix} 0 & 3 & -2 \\ -3 & 0 & 1 \\ 2 & -1 & 0 \end{bmatrix}$.

(b) Find a suitable P such that the matrix $B = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ is similar to $\begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$.