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Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, April 2022

First Degree Programme under CBCSS

Mathematics

Core Course XI

MM 1643 COMPLEX ANALYSIS II

(2014 & 2017 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – I

Answer **all** the **ten** questions are compulsory. Each question carries **1** mark.

1. Find the singular points of the function $\frac{z+1}{z^2(z-i)}$.

2. Find the power series expansion of $\frac{1}{z-4}$ in a disk of radius 1 centred at z = 5.

3. Find $\int_{|z|=2} \frac{3}{z-3} dz$.

4. Find the residue at $z = 0$ of the function $\frac{1}{z+z^2}$.

5. Find the essential singularity of $e^{1/z}$.

6. Find the order of zero of $f(z) = z^3 - 8$.

P.T.O.

7. If z_0 is a pole of a function f then what is the value of $\lim\limits_{z \to z_0} f(z)$?

8. Determine the type of singularity of $f(z) = \sin z / z$.

9. Define Isolated singular points of a complex function with an example.

10. Determine $\int\limits_{|z|<2} \frac{ze^z}{(z^2+9)^5} dz$.

(10 × 1 = 10 Marks)

SECTION – II

Answer **any eight** questions from this section. Each question carries **2** marks.

11. Find the power series expansion of $\sin(1/z)$ around $z = 1$.

12. Determine the nature of all singularities of $f(z) = \cos[1/z]$.

13. Find the residue of the function $f(z) = \tanh z / z^2$.

14. Evaluate the integral $\int\limits_{|z|=2} \tan z\, dz$.

15. Show that $\operatorname*{Res}\limits_{z=\pi i} \frac{z-\sinh z}{z^2 \sinh z} = \frac{i}{\pi}$.

16. State Jordan's lemma.

17. Show that $\int\limits_{|z|=1} \exp\left(\frac{1}{z^2}\right) dz = 0$.

18. Describe any two different types of singular points with example.

19. Show that 2 is a simple pole of $f(z) = \frac{z^2-2z+3}{z-2}$.

20. Determine the order m of each pole, the corresponding residue B for $f(z) = \left(\frac{z}{2z+1}\right)^{x^3}$.

21. Find the Cauchy principal value of the integral $\int_{-\infty}^{\infty} \frac{x \sin x \, dx}{x^2 + 2x + 2}$.

22. Define reside of a function $f(z)$ at infinity.

(8 × 2 = 16 Marks)

SECTION – III

Answer **any six** questions from this section. Each question carries **4** marks.

23. Show that $z = \pi i / 2$ is a simple pole of $f(z) = \tan z / z^2$.

24. Find

(a) $\int_{|z|=1} \sec z \, dz$

(b) $\int_{|z|=1} \frac{dz}{z^2 + 4}$.

25. Evaluate

(a) $\oint_{|z|=3} \frac{e^z}{z-2} dz$

(b) $\oint_{|z|=3} \frac{dz}{z-3i} dz$

26. Using Cauchy Integral formula, evaluate $\int_{|z|=1} \frac{e^z \cos z \, dz}{\left(z - \frac{\pi}{4}\right)^3}$

27. Let two functions p and q be analytic at a point z_0. If $p(z_0) \neq 0$, $q(z_0) = 0$, and $q'(z_0) \neq 0$, then show that z_0 is a simple pole of the quotient $p(z)/q(z)$ and $\operatorname*{Res}_{z=z_0} \frac{p(z)}{q(z)} = \frac{p(z_0)}{q(z_0)}$.

28. Show that $1+i$ is an isolated singularity of $\frac{z}{z^4 + 4}$.

29. Find the poles and residues of $f(z) = \dfrac{e^z}{z^2 + \pi^2}$.

30. Evaluate $\displaystyle\sum_1^\infty \frac{1}{n^2 + 1}$.

31. Using Cauchy Residue theorem, evaluate $\displaystyle\int_{|z|=2} \frac{\sin z}{z^6} dz$.

(6 × 4 = 24 Marks)

SECTION – IV

Answer **any two** questions. Each question carries **15** marks.

32. (a) State and prove Cauchy Residue theorem.

    (b) Using this, evaluate $\displaystyle\int_{|z|=2} \frac{5z - 2}{z(z-1)} dz$.

33. Evaluate $\displaystyle\int_0^\infty \frac{x^2}{x^6 + 1} dx$.

34. (a) Find $\displaystyle\int_0^{2\pi} \frac{d\theta}{5 + 4\sin\theta}$

    (b) Find $\displaystyle\int_0^{2\pi} \frac{d\theta}{1 + a\cos\theta}$

35. (a) State and prove Cauchy Integral formula.

    (b) Evaluate $\displaystyle\int_C \frac{3z^3 + 2}{(z-1)(z^2 + 9)} dz$ taken counter clockwise around the circle $|z - 2| = 2$ and $|z| = 4$.

(2 × 15 = 30 Marks)