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Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, April 2022

First Degree Programme under CBCSS

Mathematics

Core Course XIII

MM 1645 : INTEGRAL TRANSFORMS

(2018 & 2019 Admission)

Time : 3 Hours Max. Marks : 80

PART – A

All the **first ten** questions are compulsory. They carry **1** mark each.

1. Find the Laplace transform of $f(t) = \cos 2t$.

2. If $L[f(t)] = F(s)$, then $L[f'(t)] =$ ____________

3. Find the inverse Laplace transform of $\frac{1}{s^2 + 9}$.

4. Define unit step function.

5. Write $L\{f''(t)\}$ in terms of $L(f)$, $f(0)$ and $f'(0)$.

P.T.O.

6. Define Fourier sine transform of a function $f(x)$.

7. What is the standard form of Fourier series for an odd function?

8. If $f(x)$ and $g(x)$ have period p then find the period of $a\, f(x) + b\, g(x)$ with any constant a and b.

9. If $f(x)$ has period p then find the period of $f(nx)$.

10. Find the fundamental period of the function $\cos\left(\frac{x}{5}\right)$.

(10 × 1 = 10 Marks)

PART – B

Answer **any eight** questions. Each question carries **2** marks.

11. Find the Laplace transform of $f(t) = t \cos 4t$.

12. Find the Laplace transform of $f(t) = \cos 3t \cos 2t$.

13. Find $L\left(e^{-3t} \cos 2t\right)$.

14. Evaluate $L^{-1}\left[\frac{2}{(s+4)^3}\right]$.

15. Find $L^{-1}\left(\frac{1}{(s+1)(s+2)}\right)$.

16. Is $L[f(t)\, g(t)] = L[f(t)]\, L[g(t)]$? Explain.

17. Solve $y'' + y' - 6y = 0$, $y(0) = 1$, $y'(0) = 1$.

18. Find the convolution of t and e^{-t}.

19. Write down the Euler formulae for calculating the Fourier coefficients of function $f(x)$ of period 2π.

20. Find the Fourier series of $f(x) = x$ for $0 < x < 2\pi$.

21. Find the Fourier transform of $f(x)$ given by $f(x) = 3$ if $-2 \le x \le 2$ and $f(x) = 0$, otherwise.

22. Find the Fourier sine series for the function $f(x)$, where $f(x) = \pi - x$ in $0 < x < \pi$.

23. Derive the relation between $F[f'(x)]$ and $F[f(x)]$.

24. State the convolution theorem of Fourier transform.

25. Show that sum of two odd functions is odd.

26. Check whether the following functions are odd or even

(a) $e^{-|x|}$ (b) $x^3 \cos nx$.

(8 × 2 = 16 Marks)

PART – C

Answer **any six** questions. Each question carries **4** marks.

27. Find the Laplace transform of the function $f(t) = \begin{cases} t, & t \ge 2 \\ 0, & t < 2 \end{cases}$.

28. Find the inverse transform of $(3s-137)/(s^2+2s+401)$.

29. Solve $y'' + 3y' + 2y = r(t) = u(t-1) - u(t-2)$, $y(0) = 0$, $y'(0) = 0$.

30. Find the Laplace transform of the integral $\int_0^t t e^{-4t} \sin 3t \, dt$.

31. Find the inverse Laplace transform of $\frac{s\left(e^{-3s}-e^{-7s}\right)}{s^2+25}$.

32. Find Fourier cosine transform of $f(x)=\begin{cases}1 & 0<x<a\\ 0 & x>a\end{cases}$

33. Discuss the Fourier series representation of $f(x)=\sin\left(\frac{1}{x}\right)$ in $0<x<1$ treating $f(x)$ as a periodic function with period 1.

34. Expand the function defined by $f(x)=\begin{cases}0 & \text{for } -2<x<0\\ x & \text{for } 0\le x<2\end{cases}$ as a Fourier series on $[-2, 2]$.

35. For the function $f(x)=0$ when $x<-\pi, x>\pi$ and $f(x)=-1$ if $-\pi<x<0$ and 1 for $0<x<\pi$.

(a) Find Fourier integral representation of $f(x)$

(b) What will be the value of the integral at $x=-\pi$.

36. Show that the Fourier transform is a linear operator.

37. Express $f(x)=\begin{cases}\frac{1}{2} & \text{if } 0<x<\pi\\ 0 & \text{if } x>\pi\end{cases}$ as a Fourier sine integral.

38. Find Fourier cosine transform of $f(x)=\begin{cases}x, & 0<x<a\\ 0, & x>a\end{cases}$.

(6 × 4 = 24 Marks)

PART – D

Answer **any two** questions. Each question carries **15** marks.

39. (a) If $L[f(t)] = F(s)$ Show that $L[f(t-a)u(t-a)] = e^{-as} F(s)$.

(b) Using Laplace transform solve $y''+4y'+3y=e^{-t}$, $y(0)=y'(0)=1$.

40. (a) State the convolution theorem for Laplace transforms.

(b) Use it to find the inverse Laplace transform of $\dfrac{s}{(s-1)(s^2+4)}$. Verify the result by finding the inverse using partial fraction technique.

41. If $f(x)=e^{-kx}$ $(x>0, k>0)$. Then

(a) Find the Fourier sine and cosine transform of $f(x)$

(b) Prove that $\int_0^\infty \dfrac{x \sin mx}{x^2+k^2} dx = \dfrac{\pi}{2} e^{-km}$.

42. Find a Fourier series to represent $f(x)=x-x^2$ in $(-\pi, \pi)$ and hence show that $\dfrac{1}{1^2} - \dfrac{1}{2^2} + \dfrac{1}{3^2} - \dfrac{1}{4^2} + \ldots = \dfrac{\pi^2}{12}$.

43. (a) Represent $f(x) = e^{-kx}$ $(x>0, k>0)$ as a Fourier cosine integral

(b) Find the Fourier transform of $f(x)$, where

$$f(x) = \begin{cases} 1 & \text{if } |x| < 1 \\ 0 & \text{otherwise} \end{cases}$$

44. (a) Obtain the half range Fourier cosine series for the function

$$f(x)=\begin{cases}\cos x, & 0 < x < \pi/2 \\ 0, & \pi/2 < x < \pi\end{cases} \text{ in } (0, \pi).$$

(b) Find a Fourier series that represents $f(x) = |x|$ in $[-\pi, \pi]$ and deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \ldots = \frac{\pi^2}{8}$.

(2 × 15 = 30 Marks)
