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Reg. No. : ....

Name : ....

Sixth Semester B.Sc. Degree Examination, March 2020

First Degree Programme Under CBCSS

Mathematics

Core Course X

MM 1642 : Linear Algebra

(2014 Admission Onwards)

(Special Examination)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the first ten questions are compulsory. **Each** question carries **1** mark.

1. Find the line through $B = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ parallel to the vector $A = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$.

2. Write the polar form of $X = \begin{pmatrix} 3 \\ 0 \end{pmatrix}$.

3. Find the vector, which is perpendicular to the vector $\begin{pmatrix} x \\ y \end{pmatrix}$.

4. Find the angle between $A = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

5. Find $S\begin{pmatrix} 0 \\ 10 \end{pmatrix}$ if S is the transformation which assigns to each vector X the reflection of X in the line along the vector $\begin{pmatrix} 1 \\ 2 \end{pmatrix}$.

P.T.O.

6. Find a transformation which is equal to $R_{\frac{3\pi}{2}}$, which rotates each vector counterclockwise by $\frac{3\pi}{2}$ radians.

7. Let P be denote projection on the y–axis and let L be the x-axis. Find the image of L under P.

8. Find the eigen values of the matrix $A = \begin{pmatrix} 3 & 0 \\ 0 & -5 \end{pmatrix}$.

9. Define an isometry transformation.

10. Find two orthogonal vectors in $\mathbb{R}^4$.

SECTION – II

Answer any **eight** questions from this section **Each** question carries **2** marks.

11. Find the projection of $B = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$ to the line along $A = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$.

12. Find the distance from $\begin{pmatrix} 3 \\ 1 \end{pmatrix}$ to the line $y = 2x$.

13. Find $R_{\pi/2}(X)$ where $X = \begin{pmatrix} x \\ y \end{pmatrix}$.

14. Find the matrix of the linear transformation D_r, where D_r is the stretching by *r.*

15. Describe the parallelogram determined by the vectors $\begin{pmatrix} 2 \\ 1 \end{pmatrix}$ and $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

16. Exhibit a linear transformation N such that $N \neq 0$, while $NN = 0$.

17. Show that multiplying a matrix on the left by a permutation matrix interchanges the rows while the corresponding multiplication on the right interchanges the columns.

18. Prove that inverse of a linear transformation if it exists is unique.

19. Let P be the projection on the x–axis. Prove that P has no inverse.

20. Find all solutions of the system

$$x + 2y = 3$$
$$-2x - 4y = -6$$

21. Let E be a linear transformation such that $E^2 = E$. What are the eigen values of E?

22. Let K be reflection in the x_1x_2–plane and J be reflection in the x_2x_3–plane. Prove that $KJ = JK$.

SECTION – III

Answer any **six** questions from this section. **Each** question carries **4** marks.

23. B is reflection in the x–axis and A is reflection in the y–axis. Prove that BA is rotation by π radians.

24. Let L be a straight line through the origin and denote by θ the angle from the positive x–axis to L. Find the matrix of the transformation S which reflects each vector in L.

25. Let $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$ be an arbitrary matrix. Prove that we can find elementary matrices e_1, e_2, e_3, e_4 and a diagonal matrix $\begin{pmatrix} d_1 & 0 \\ 0 & d_2 \end{pmatrix}$ such that $e_1e_2\begin{pmatrix} a & b \\ c & d \end{pmatrix}e_3e_4 = \begin{pmatrix} d_1 & 0 \\ 0 & d_2 \end{pmatrix}$.

26. For what choices of the numbers u, v does the system

    $x + y = u$

    $5x + 5y = v$

    have a solution?

27. Find all solutions of the non-homogeneous system

    $x_1 + x_2 + x_3 = 1$

    $2x_1 - x_2 = 5$

    $5x_1 + 2x_2 + 3x_3 = 8$

28. Let T be the linear transformation with matrix

    $$m(T) = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

    (a) Show that $T^2 = 0$

    (b) Determine the eigen values of T

    (c) Find all eigen vectors of T.

29. If T is an isometry of $\mathbb{R}^3$, prove that T has 1 or –1 as an eigenvalue and has no other eigenvalues.

30. Let m be a symmetric 3 × 3 matrix and let M be the corresponding linear transformation. Let t_1, t_2 be distinct eigenvalues of M and let X_1, X_2 be corresponding eigen vectors. Prove that $X_1 \cdot X_2 = 0$.

31. Let S and T be two linear transformations. Prove that the product ST has an inverse, then both S and T has inverse.

SECTION – IV

Answer any **two** questions from this section. **Each** question carries **15** marks.

32. Let A be a linear transformation with matrix $\begin{pmatrix} a & b \\ c & d \end{pmatrix}$. Prove that A has an inverse iff $ad - bc = 0$.

33. Find all eigenvalues and corresponding eigenvectors for each of the following transformations in the plane.
    (a) Rotation by π radians
    (b) Stretching D_r
    (c) I, identity transformation.

34. Show that two vectors A and B in $\mathbb{R}^3$ are linearly dependent iff $A \times B = 0$.

35. Let T be a transformation of 3–space. Prove that T is a linear transformation iff T satisfies the conditions
    $T(X+Y) = T(X) + T(Y)$ and
    $T(rX) = rT(X)$.
    for all vectors X and Y and all scalars r.