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Reg. No. : ....

Name : ....

Third Semester M.Sc. Degree Examination, February 2021

Mathematics

MM 232 : FUNCTIONAL ANALYSIS — I

(2017 Admission onwards)

Time : 3 Hours Max. Marks : 75

(Answer either part A or B of each question)

I. (A) (a) Show that l^1 is a proper subspace of c_0. **4**

(b) Show that if X is a finite dimensional normed space, then every closed and bounded subset of X is compact. **7**

(c) Is $(\mathbb{R}^2, \| \|_1)$ strictly convex? Justify. **4**

(B) (a) Show that every bijective linear map from a finite dimensional normed space X to a normed space Y is a homeomorphism. **4**

(b) Show that a linear functional f on a normed space X is continuous if and only if the zero space $Z(f)$ is closed in X. **7**

(c) Give an example of a discontinuous linear functional. **4**

II. (A) (a) State and and prove Hahn-Banach separation theorem. **7**

(b) State the Hahn-Banach extension theorem. Is the Hahn-Banach extension unique? Justify. **6**

(c) Give an example of a convex set which is not a convex body. **2**

P.T.O.

(B) (a) Show that if Y is a proper dense subspace of a Banach space X, then Y is not a Banach space in the induced norm. **4**

(b) Prove that a Banach space cannot have a denumerable basis. **7**

(c) Is $\left(c_{00}, \| \|_2\right)$ a Banach space? Justify. **4**

III. (A) (a) Show that a set of continuous functions from a metric space to a metric space can be bounded at each point without being uniformly bounded. **5**

(b) State the Uniform boundedness principle and explain it geometrically. **5**

(c) Show that the hypothesis of completeness on X cannot be dropped from the Uni-form boundedness principle. **5**

(B) (a) Show that a continuous map is always closed. **2**

(b) State and prove closed graph theorem. **9**

(c) State the open mapping theorem and show that the completeness of X and Y cannot be dropped. **4**

IV. (A) (a) State and prove Bounded inverse theorem. **4**

(b) State and prove two norm theorem. **11**

(B) (a) If A is an invertible bounded linear operator on a normed space X, then show that $\sigma(A^{-1}) - \{k^{-1} : k \in \sigma(A)\}$. **4**

(b) State and prove spectral radius formula. **11**

V. (A) (a) Show that weak convergence and convergence are equivalent in a finite dimensional normed space. **7**

(b) Prove that a closed subspace of a reflexive normed space is reflexive. **8**

(B) (a) Show that not every continuous linear map is compact. **3**

(b) If X is a normed space and Y is a Banach space, then show that $CL(X, Y)$ is a closed subspace of $BL(X, Y)$. **9**

(c) Is the map given by $F(x_1, x_2) = (x_1 + x_2, x_2)$ compact? Justify. **3**