(Pages : 7) **K – 4879**

Reg. No. : ....

Name : ....

Third Semester M.Sc. Degree Examination, February 2021.

Mathematics

MM 233 – Elective I : OPERATIONS RESEARCH

(2013 Admission Onwards)

Time : 3 Hours Max. Marks : 75

Answer **any one** questions from each **Unit**.

Unit – I

1. (a) Write down the standard form of a linear programming problem.

(b) Solve the following LP problem graphically

Maximize : $z = 6x_1 + 5x_2$

Subject to : $3x_1 + x_2 \leq 160$

$x_1 \leq 40$

$x_2 \leq 130$

$x_1 \geq 80$

$x_1, x_2 \geq 0$

(c) Consider a system of two equations in five unknown as follows:

$$x_1 + 2x_2 + 10x_3 + 4x_4 = 5$$
$$x_1 + x_2 + 4x_3 + 3x_4 = 8$$
$$x_1, x_2 ...., x_5 \geq 0$$

P.T.O.

(i) Reduce the system to canonical form with respect to (x_1, x_2) as basic variables. Write down the basic solution. Is it feasible. why or why not?

(ii) What is the maximum number of basic solutions possible?

2. (a) Explain the following terms

(i) Feasible solution

(ii) Feasible region

(iii) Optimal solution

(iv) Unbounded solution

(b) Transform the following LP problems to the standard form

Minimize : $z = -3x_1 + 4x_2 - 2x_3 + 5x_4$

Subject to : $4x_1 - x_2 + 2x_3 - x_4 = -2$

$$x_1 + x_2 + 3x_3 - x_4 \le 14$$
$$-2x_1 + 3x_2 - x_3 + 2x_4 \ge 2$$
$$x_1 \ge 0, x_2 \ge 0, x_3 \le 0, x_4 \text{ unrestricted in sign.}$$

(c) Use the simplex method to solve

Maximize : $z = x_1 + 3x_2$

Subject to : $x_1 \le 5$

$$x_1 + 2x_2 \le 10$$
$$x_2 \le 4$$
$$x_1, x_2 \ge 0$$

Unit – II

3. (a) Explain the least cost rule.

   (b) A batch of four jobs can be assigned to five machines. The setup time for each job on various machines is given by the following table

		Machine				
		1	2	3	4	5
	1	10	11	4	2	8
Job	2	7	11	10	14	12
	3	5	6	9	12	14
	4	13	15	11	10	7

   Find an optimal assignment of jobs to machines which will minimize the total setup time.

   (c) What are the various types of transportation problems? Discuss with suitable examples.

4. (a) How do you convert an unbalanced transportation problem into a standard LP problems?

   (b) Explain any two methods for finding initial feasible solution in a transportation problem.

   (c) Four different jobs can be done on four different machines. The set up and take down time costs are assumed to be prohibitively high for changeovers. The matrix below gives the cost in rupees of producing jobs *i* on machine *j*.

		Machines			
		M1	M2	M3	M4
	J1	16	10	14	11
Job	J2	14	11	15	15
	J3	15	15	13	12
	J4	13	12	14	15

   How should the jobs be assigned to the various machines so that the total cost is minimized? Represent the problem as an LP problems.

Unit III

5. (a) What are the essential difference between PERT and CPM?

(b) Consider a project consisting of nine jobs $(A, B, \ldots I)$ with the following precedence relations and time estimates:

Job	Predecessor	Time (days)
A	-	15
B	-	10
C	A,B	10
D	A,B	10
E	B	5
F	D,E	5
G	C,F	20
H	D,E	10
I	G,H	15

(i) Draw the project network for this problem designating the jobs by arcs and events by nodes.

(ii) Determine the earliest completion time of the project, and identify the critical path.

(iii) Determine a project schedule listing the earliest and latest starting times of each job. Also identify the critical job.

6. (a) Draw the network for a project consisting of 16 jobs $A, B, C, D, \ldots M, N, O, P$ with the following job sequence.

$A, B, C, D \to E, F, G$

$E, F, G \to H$

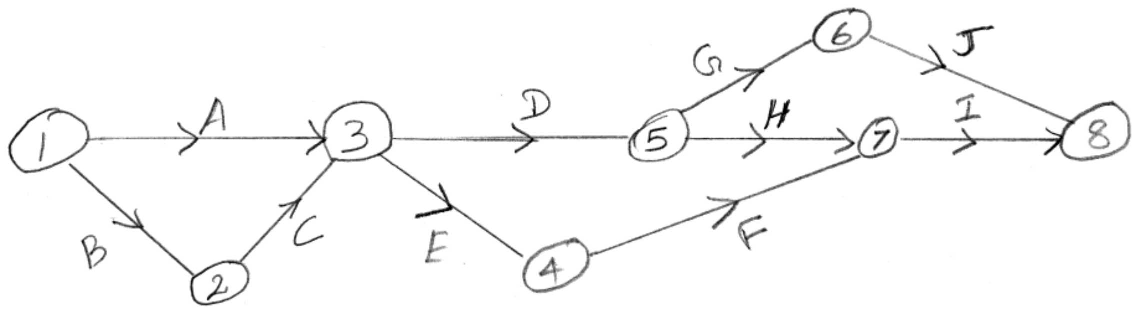
$H \to I$

$I \to J, K, L, M, N$

$J, K, L, M, N \to O$

$G, O \to P$

(b) Consider the project network given below



The data for normal times, crash times, and crashing costs are given as follows:

Job	Normal time (Days)	Crash Time (Days)	Cost of crashing for day ($)
A	10	7	4
B	5	4	2
C	3	2	2
D	4	3	3
E	5	3	3
F	6	3	5
G	5	2	1
H	6	4	4
I	6	4	3
J	4	3	3

Let T represent the earliest completion time of the project

(i) Determine the maximum and minimum value of T

(ii) Set up the linear program to solve the CPM problems if the project is to be completed in 21 days at minimum cost.

(iii) Given the overhead cost as $ 5 per day, determine the optimal duration of the project in terms of both the crashing and the overhead costs by direct enumeration method.

Unit IV

7. (a) Write the Kuhn–Tucker conditions for minimizing a non linear programming problem.

(b) Solve the following problems by K.T. conditions and verify geometrically:

(i) Minimize x_1,

(ii) Maximize x_2 in each case,

subject to $(x_1 - 4)^2 + x_2^2 \leq 16$,

$$(x_1 - 3)^2 + (x_2 - 2)^2 = 13$$

8. (a) Write a short note on Quadratic programming.

(b) Show that in the problem

Maximize $3x_1 + 6x_2 - 4x_1x_2 - 3x_1^2 - 2x_2^2$

Subject to $3x_1 + 2x_2 \leq 4$,

$x_1 + x_2 \leq 1$,

$x_1, x_2 \geq 0$ the objective function is strictly concave. Hence solve the problem by quadratic programming algorithm.

Unit V

9. (a) What is meant by backward and forward recursion.

(b) Solve by forward recursion
Min $u_1^2 + u_2^2 + u_3^2$
Subject to $u_1 + u_2 + u_3 \geq 10$
$u_1, u_2 \geq 0$

10. (a) Explain "The minimum path problem".

(b) Find the shortest path from a vertex in $j=0$ to a vertex in $j=4$, movement always being from j to $j+1$, in the following graph.