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Reg. No. : ....

Name : ....

Third Semester M.Sc. Degree Examination, February 2021

Mathematics

MM 234 : ELECTIVE II — GRAPH THEORY

(2017 Admission onwards)

Time : 3 Hours Max. Marks : 75

Answer either **Part A** or **Part B** of each question. All question carry equal marks.

I. (A) (a) Define automorphism group Aut(G) of a graph G. Check whether Aut(G) = Aut(G^c) where G^c is the complement of the graph G. Justify your answer.

(b) Determine Aut(C_5).

(B) (a) For every graph G, prove that $K(G) \leq \lambda(G) \leq \delta(G)$.

(b) Prove that a 3-regular graph has a cut vertex if and only if G has a bridge.

II. (A) (a) Define Eulerian graph. Prove that a connected graph G contains an Eulerian trial if and only if exactly two vertices of G have odd degree.

(b) Find all positive integers n such that the complete graph K_n is Eulerian. Find a necessary and sufficient condition for the cartesian product G $\times$ H of two nontrivial connected graphs G and H to be Eulerian. Justify your findings.

(B) (a) State and prove Ore's theorem.

(b) Define h(G) and h*(G) of a graph G. Prove that for every connected graph G, h(G) = h*(G).

P.T.O.

III. (A) (a) Define digraph with example. State and prove a necessary and sufficient condition for a diagraph to be a strong digraph.

(b) Define tournament and transitive tournament with example. Prove that a Tournament T is transitive if and only if T has no cycles.

(B) (a) Define Edge covering number (α_1) and Edge independence number (β_1) of a graph G. Prove that for every graph G of order n containing no isolated vertices, $\alpha_1(G)+\beta_1(G)=n$.

(b) Define 2-factor of a graph G. Prove that a graph G is 2-factorable if and only if G is r-regular for some positive even integer *r*.

IV. (A) (a) Define chromatic number of a graph G. For every graph G Prove that $\chi(G)\leq 1+\max\{\delta(H)\}$ where maximum is taken over all induced subgraphs H of G.

(b) Define chromatic index of a graph G. State and prove König's theorem.

(B) State and prove Turán's theorem.

V. (A) (a) Define boundary vertex of a graph G. Prove that no cut vertex of a connected graph G is a boundary vertex of G.

(b) Define complete vertex of a graph G. Prove that a vertex v of a connected graph G is a boundary vertex of every vertex distinct from v if any only if v is a complete vertex of G.

(B) (a) Define locating set and location number of a graph G. Prove that a connected graph G of order n has locating number 1 if and only if $G\cong P_n$.

(b) Define Detour distance. Prove that detour distance is a metric on the vertex set of every connected graph and that $rad_D(G)\leq diam_D(G)\leq 2\,rad_D(G)$.
