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Reg. No. : ....

Name : ....

First Semester B.Sc. Degree Examination, November 2019

First Degree Programme under CBCSS

Complementary Course I for Chemistry and Polymer Chemistry

MM 1131.2 : MATHEMATICS I – CALCULUS WITH APPLICATIONS IN CHEMISTRY – I

(2018 Admissions Onwards)

Time : 3 Hours Max. Marks : 80

PART – I

Answer **all** questions. **Each** question carries **1** mark.

1. Find the derivative of $\frac{1}{x^3+2x-3}$ with respect to r.

2. Use implicit differentiation to find $\frac{dy}{dx}$ if $4x^2-2y^2=9$.

3. Find the argument of the complex number $z=2-3i$.

4. Give the polar form of the complex number $2i$.

5. Find the conjugate of the complex number $z=(x+5i)^{(3y+2ix)}$.

6. Find $\frac{dy}{dx}$ where $y=\cosh(x^3)$.

P.T.O.

7. Evaluate $\int xe^3 dx$.

8. Two particles have velocities $\vec{v}_1 = \hat{i} + 3\hat{j} + 6\hat{k}$ and $\vec{v}_2 = \hat{i} - 2\hat{k}$ respectively. Find the velocity $\vec{u}$ of the second particle relative to the first.

9. Find a unit vector normal to both the vectors $\vec{a} = \hat{i} + 2\hat{j} - 3\hat{k}$ and $\vec{b} = -4\hat{i} + \hat{j} + 2\hat{k}$.

10. Give the equation of the line through the fixed point A with position vector $\vec{a}$ and having a direction $\vec{b}$.

PART – II

Answer any **eight** questions. **Each** question carries **2** marks.

11. Find the third derivative of the function $f(x) = x^2 \cos x$, using Leibnitz' theorem.

12. Verify Rolle's theorem for the function $f(x) = x^2 + 6x$ on $[-6, 0]$.

13. Find $f'(x)$ if $f(x) = \sqrt{4 + \sqrt{3x}}$.

14. Find the interval on which $f(x) = 5 + 12x = x^3$ is increasing.

15. Find the general value of $Ln(-i)$.

16. Express $z = \dfrac{x - 2i}{-1 + 4i}$ in the form $x + iy$.

17. Prove that $\cosh x - \cosh y = 2\sin\left(\dfrac{x+y}{2}\right)\sinh\left(\dfrac{x-y}{2}\right)$.

18. Evaluate $\int_0^2 (2 - x)^{-1/4} dx$.

19. Find the mean value of the function $f(x) = x^2$ over $[-1, 2]$.

20. Find the angle between the vectors $\vec{a} = \hat{i} - 2\hat{j} + 2\hat{k}$ and $\vec{b} = -3\hat{i} + 6\hat{j} - 6\hat{k}$.

21. Find the area of a triangle which is determined by the points $P_1(2,0,-3)$, $P_2(1,4,5)$ and $P_3(7,2,9)$.

22. Find an equation of the line in space that passes through the points $P_1(2,4,-1)$ and $Q(5,0,7)$.

PART – III

Answer any **six** questions. **Each** question carries **4** marks.

23. Find the positions and natures of the stationary points of the function $f(x)=x^3-3x^2+3x$.

24. Show that the radius of curvature at the point (x,y) on the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ has magnitude $\frac{(a^4y^2+b^4x^2)^{3/2}}{a^4b^4}$ and the opposite sign to y.

25. Verify Mean Value theorem for $f(x)=x^3+x-4$ in $[-1,2]$. Find all values of c in that interval which satisfy the conclusion of the theorem.

26. Express $\cos 4\theta$ in powers of $\cos\theta$ and $\sin\theta$.

27. Prove that $\cos^7\theta=\frac{1}{64}(\cos 7\theta+7\cos 5\theta+21\cos 3\theta+35\cos\theta)$.

28. Find all values of $(-1)^{1/6}$.

29. Find the volume of the parallelepiped with sides $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$, $\vec{b}=4\hat{i}+5\hat{j}+6\hat{k}$ and $\vec{c}=7\hat{i}+8\hat{j}+10\hat{k}$.

30. Find the minimum distance from the point P with sides coordinates $(1,2,1)$ to the line $\vec{r}=\vec{a}+\lambda\vec{b}$, where $\vec{a}=\hat{i}+\hat{j}+\hat{k}$ and $\vec{b}=2\hat{i}-\hat{j}+3\hat{k}$.

31. A line is given by $\vec{r}=\vec{a}+\lambda\vec{b}$, where $\vec{a}=\hat{i}+2\hat{j}+3\hat{k}$ and $\vec{b}=4\hat{i}+5\hat{j}+6\hat{k}$. Find the coordinates of the point P at which the line intersects the plane $x+2y+3z=6$.

PART – IV

Answer any **two** questions. **Each** question carries **15** marks.

32. (a) At what point does $y = e^x$ have maximum curvature? (7)

(b) Solve the equation $z^6 + z^4 + z^2 + 1 = 0$. (8)

33. (a) If $I_n = \int_0^{\pi/2} \sin^n x dx$ and if n is any positive integer, show that $I_n = \frac{n-1}{n} I_{n-2}$.

Hence evaluate $\int_0^{\pi/2} \sin^6 x dx$. (7)

(b) Find the area and perimeter of the Cardiod $\rho = 1 - \cos\phi$. (8)

34. (a) Find the area of the surface that is generated by revolving the portion of the curve $y = x^2$ between $x = 1$ and $x = 2$ about the $y-axis$. (7)

(b) Find an equation of the plane that contains the line $x = -2 + 3t, y = 4 + 2t, z = 3 - t$ and is perpendicular to the plane $x - 2y + z = 5$. (8)

35. (a) Find an equation of the plane whose points are equidistant from $(2, -1, 1)$ and $(3, 1, 5)$. (5)

(b) Find the radius ρ of the circle that is the intersection of the plane $\hat{n}.\vec{r} = p$ and the sphere of radius a centered on the point with position vector $\vec{c}$. (10)
