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Reg. No. : ....

Name : ....

Third Semester B.Sc. Degree Examination, January 2023

First Degree Programme under CBCSS

Statistics

Complementary Course for Physics

ST 1331.2 : PROBABILITY DISTRIBUTIONS AND STOCHASTIC PROCESS

(2017 and 2018 Admission)

Time : 3 Hours

Max. Marks : 80

SECTION – A

Answer **all** questions. Each carries **1** mark.

1. X is binomially distributed with parameters n and p. What is the distribution of $Y = n - x$?

2. Name the discrete distribution for which variance is greater than the mean.

3. What is the mode of a normal distribution $N(\mu, \sigma)$?

4. Name the continuous distribution which possess lack of memory property.

5. The Beta distribution of first kind reduces to ———————— distribution when both the parameters equal to 1.

6. What is the variance of Chi square distribution with 2 degrees of freedom?

7. Define F statistic.

P.T.O.

8. What is the mode of a random variable follow t distribution with n degrees of freedom?

9. What do you mean by ordered samples?

10. Define state space.

(10 × 1 = 10 Marks)

SECTION – B

Answer any **eight** questions. Each carries **2** marks.

11. Define negative binomial distribution.

12. Show that the mode of the distribution $p(x)=\left(\frac{1}{2}\right)^x; x = 1,2,3,\cdots$ is 1.

13. Show that for a Poisson distribution the coefficient of variation is the reciprocal of the standard deviation.

14. Obtain a recurrence relation for probabilities of a Binomial distribution.

15. Define Weibull distribution.

16. State additive property of Normal distribution.

17. Find the mean of beta distribution of first kind.

18. Define statistic and sampling distribution

19. If $F \sim F(m,n)$ find the distribution of $\frac{1}{F}$?

20. Define Maxwell-Boltzmann statistic.

21. Define transition probability matrix.

22. Define process with independent increments.

(8 × 2 = 16 Marks)

SECTION – C

Answer any **six** questions. Each carries **4** marks.

23. Prove the recurrence relation between the central moments of Poisson distribution $\mu_{r+1} = \lambda\left(r\mu_{r-1} + \frac{d\mu_r}{d\lambda}\right)$, where μ_r is the r^{th} moment about the mean λ.

24. Obtain the moment generating function of Binomial distribution with $n = 7$ and $p = 0.6$. Find the first three central moments of the distribution.

25. A fair coin is tossed three times. Find the probability that (1)3 tails occur (2)2 tails and 1 head occur.

26. If X has a uniform distribution in $[0,1]$ find the distribution of $-2\log X$. Identify the distribution also.

27. X is normally distributed with mean 12 and SD. 4. Find out the probability of the following:

    (a) $P(X \geq 20)$

    (b) $P(0 \leq X \leq 12)$

28. Let X and Y be independent standard normal variates. State the distribution of X^2 / Y^2 and write down its pdf.

29. Explain the relation between normal, chi square, t and F distributions.

30. Define Chi square distribution and state its uses.

31. Explain the classification of stochastic processes.

(6 × 4 = 24 Marks)

SECTION – D

Answer any **two** questions. **Each** carries **15** marks.

32. Derive Poisson distribution as a limiting form of a binomial distribution.

33. Define Normal distribution. Derive mgf of normal distribution and hence find mean and variance.

34. Derive the sampling distribution of mean of samples taken from a normal population.

35. Fit a normal distribution to the following data:

Class:	21-24	25-28	29-32	33-36	37-40
f:	4	8	12	10	6

(2 × 15 = 30 Marks)

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Reg. No. : ....

Name : ....

Third Semester B.Sc. Degree Examination, January 2023

First Degree Programme Under CBCSS

Statistics

Complementary Course for Psychology

ST 1331.5 : STATISTICAL METHODS FOR PSYCHOLOGY III

(2019 Admission Onwards)

Time : 3 Hours Max. Marks : 80

(Use of Statistical Table and Calculator are permitted)

SECTION – A

Answer **all** questions. **Each** question carries **1** mark.

1. The limits of the correlation coefficient is ______________
2. The diagram using for the identification of relationship between variable is ______________
3. What is meant by regression?
4. If the variables X and Y are independent, then the value of the regression coefficient is ______________
5. For mutually exclusive event A and B, $P(AUB)$is ______________
6. What is probability density function?
7. What is complete association?
8. What is the probability of getting at least one head in the experiment of tossing two coins?

P.T.O.

9. Define Z score.

10. What is kurtosis?

(10 × 1 = 10 Marks)

SECTION – B

Answer **any eight** questions. **Each** question carries **2** marks.

11. Distinguish between direct and inverse correlation.

12. Define Karl Pearson's correlation coefficient.

13. What is the use of scatter diagram?

14. What do you mean by curve fitting?

15. Explain stanine score.

16. What is the mean of Poisson distribution?

17. Write down the significance of the study of regression.

18. Define discrete random variable with an example?

19. If X is binomially distributed with parameter n and p. What is the distribution of $Y = n - X$?

20. Examine whether $f(x)$ defined below is a pdf.

$$f(X) = \begin{cases} 0, & x < 2 \\ \dfrac{3-2x}{18}, & 2 \leq x < 4 \\ 0\ , & x > 4 \end{cases}$$

21. What are the two types of random variables?

22. Distinguish between association and dissociation.

23. What are the conditions of consistency for two attributes A and B?

24. What is meant by the consistency of the data?

25. What are regression lines? Why are there two regression lines?

26. What are the characteristics of skewness?

(8 × 2 = 16 Marks)

SECTION – C

Answer **any six** questions. **Each** question carries **4** marks.

27. Find the correlation coefficient for the following data:

X	6	2	10	4	8
Y	9	11	5	8	7

28. What are the different types of correlation? Explain briefly.

29. Let X and Y be Independent binomial variates, each with parameter n and p. Find $P(X-Y=K)$.

30. Prove that regression coefficients are independent of the change of origin but not of scale.

31. The two regression line are $3X-2Y=26$ and $6X+3Y=31$. Find the correlation coefficient.

32. What is scatter diagram? Draw scatter diagram when the correlation coefficient $r=+1$ and $r=-1$.

33. Define Yule's coefficient of association?

34. Write briefly about the different methods used to check consistency.

35. The mean and variance of binomial distribution are 4 and $\frac{4}{3}$ respectively. Find $P(X \geq 1)$.

36. If 5% of the people in an area are found to be mental disorders, use Poisson distribution to find the probability that in a sample of 100 people

    (a) none is having mental disorder

    (b) 5 will have mental disorder.

37. Derive the mean and variance of binomial distribution.

38. Explain the method of converting raw score into standard normalized score.

(6 × 4 = 24 Marks)

SECTION – D

Answer **any two** questions. **Each** question carries **15** marks.

39. (a) Calculate the rank correlation coefficient for the following data.

X	88	72	95	60	35	46	52	58	30	67
Y	65	90	86	72	30	54	38	43	48	75

(b) State and prove that correlation coefficient is independent of origin and scale.

40. Calculate regression coefficients and obtain the lines of regression for the following data.

X	1	2	3	4	5	6	7
Y	9	8	10	12	11	13	14

41. The mean height of soldiers to be 68.22 inches with a variance of 10.8 inches. How many soldiers in a regiment of 1000 would you expect to be, (a) over 72 inches, and (b) below 66 inches. Assume height is normally distributed.

42. Among the adult population of a certain town 50% are males. 60% are wage earners and 50% are 45 years of age or over,10% of the males are not wage earners and 40% of the males are under 45. Make the best possible inference about the limits within which the percentage of persons(male or female) of 45 years or over are wage earners.

43. Evaluate the distribution function and calculate F(2) and F(1)

$$f(X)=\begin{cases} \frac{x}{2} & ,0<x\le 1 \\ \frac{3-x}{2} & ,1<x\le 2 \\ \frac{1}{4} & ,2<x\le 3 \\ \frac{4-x}{4} & ,3<x\le 4 \\ 0 & ,elsewhere \end{cases}$$

44. Explain the characteristics and applications of Normal curve.

(2 × 15 = 30 Marks)

(Pages : 6) **P – 3909**

Reg. No. : ....

Name : ....

Third Semester B.Sc. Degree Examination, January 2023

First Degree Programme under CBCSS

Statistics

Core Course

ST 1341 : PROBABILITY AND DISTRIBUTION – I

(2019 Admission Onwards)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions. Each question carries **1** mark.

1. Define random experiment.

2. Write down the sample space for the random experiment of tossing a coin three times.

3. State multiplication theorem on probability.

4. If $V(X)=1.5$ what is $V(3X+1)$?

5. Let a random variable X takes values –3, 0, 1 and 3 with respective probabilities 0.25, 0.33. 0.12 and 0.30. Write down the distribution of X^2.

6. Define probability generating function.

7. Give the relation between MGF and cumulant generating function.

P.T.O.

8. Define mathematical expectation.

9. Suppose A and B events with $P(A) = 0.6$, $P(B) = 0.3$ and $P(AB) = 0.2$, What is the probability of neither A nor B occurs?

10. Define correlation in terms of random variables X and Y in a bivariate distribution.

(10 × 1 = 10 Marks)

SECTION – B

Answer any **eight** questions. Each question carries **2** marks.

11. Define axiomatic approach to probability. What is its importance?

12. Let a class contains 10 men and 20 women of which half the men and halt the women have brown eyes. Find the probability that person chosen at random is a man or has brown eyes.

13. Given $P(A) = \frac{1}{3}$, $P(B) = \frac{1}{4}$, and $P(A/B) = \frac{1}{8}$. What is $P(B/A)$?

14. Distinguish between discrete and continuous random variables.

15. A random variable X has the following probability function :

X :	–2	1	2	4
$P(x)$:	1/4	1/8	1/2	1/8.

Find the cumulative distribution function.

16. Prove (a) Probability of impossible event is zero and (b) $P(A^c) = 1 - P(A)$.

17. Let $f(x, y) = K$, for $0 < x < 1$, $x < y < x+1$ and $f(x, y) = 0$, otherwise. Find the value of K.

18. Define characteristic function. State the reason why it always exists.

19. Define (a) conditional expectation and (b) conditional variance in continuous case.

20. Let $f(x, y) = e^{-(x+y)}$, $0 < x < \infty$, $0 < y < \infty$ be the joint pdf of (X, Y). Examine whether X and Y are independent or not.

21. A random variable X has the probability mass function $P(x) = \frac{1}{3}\left(\frac{2}{3}\right)^x$, $x = 0, 1, 2 \ldots$ Find the mean of X.

22. Show that variance is not independent of change of scale.

23. Distinguish between probability density function and probability mass function.

24. Find the MGF of a random variable X having pdf $f(x) = 1/\theta$, $0 \le x \le \theta$ and $f(x) = 0$, otherwise.

25. A single card is drawn from an ordinary pack of 52 cards. Find the probability that the card is a red face card.

26. How to find median in case of a continuous probability distribution?

(8 × 2 = 16 Marks)

SECTION – C

Answer any **six** questions. Each question carries **4** marks.

27. Define the following terms with examples :

    (a) event,

    (b) exhaustive events,

    (c) equally likely events , and

    (d) mutually exclusive events

28. Distinguish between classical and empirical definitions of statistics.

29. The pdf of random variable X is $f(x) = 30x^2(1-x)^2$, $0 < x < 1$ and $f(x) = 0$, otherwise. Find the distribution of $Y = X^2$.

30. State and prove addition theorem on probability.

31. In a certain college, 25 percent of the students failed in mathematics, 15 percent failed in chemistry, and 10 percent failed in both mathematics and chemistry. A student is selected at random. If the student failed in chemistry, what is the probability that the student fail in mathematics? What is the probability that student fail in chemistry if the student has failed in mathematics?

32. If X and Y are random variables, show that $[E(XY)]^2 \leq E(X^2)E(Y^2)$.

33. Distinguish between Pairwise and mutually independent events.

34. Let (X, Y) have a joint pdf :

$f(x, y) = x^2 + \frac{xy}{3}$, $0 \leq x \leq 1, 0 \leq y \leq 2$ and $f(x, y) = 0$, otherwise, find $P(X < Y)$.

35. Define (a) conditional distributions and (b) marginal distributions and (c) independence of random variables.

36. If $\phi_X(t)$ is the characteristics function of a random variable X, prove the following :

(a) $\phi_X(t)$ and $\phi_X(-t)$ are conjugate functions

(b) $\phi_X(t)$ is a real valued and even function of t if the random variable X is symmetrical about zero.

37. Let $f(x, y) = 8xy, 0 < x < y < 1$ and $f(x, y) = 0$, otherwise. Find conditional expectation of Y given X.

38. Two unbiased dice are thrown. Find the expected value of the sum of the points on them.

(6 × 4 = 24 Marks)

SECTION – D

Answer any **two** questions. **Each** question carries **15** marks.

39. (a) State and prove Bayes theorem on probability.

(b) Three machines A, B and C produce, respectively 40 percent, 10 percent and 50 percent of the items in a factory. The percentage of defective items produced by the machines is, respectively, 2 percent, 3 percent and 4 percent. An item from the factory is selected at random.

(i) Find the probability that the item is defective

(ii) If the item is defective, find the probability that the item was produced by (1) machine A, (2) machine B and (3) machine C. **6 + 9 = 15**

40. (a) Define distribution function. State its properties.

(b) Let X be a continuous random variable with probability density function :

$f(x) = 6x(1-x)$ *if* $0 \le x \le 1$ and $f(x) = 0$, otherwise.

Find $P\left(X \le \frac{1}{2} \Big/ \frac{1}{3} \le X \le \frac{2}{3}\right)$

(c) State and prove addition and multiplication theorems on expectation in discrete case. **4 + 4 + 7 = 15**

41. The joint distribution of (X, Y) is given below :

$P(1, 1) = 0.1$, $P(2, 1) = 0.1$, $P(3, 1) = 0.2$, $P(1, 2) = 0.2$, $P(2, 2) = 0.3$ and $P(3, 2) = 0.1$.

Find (a) marginal distributions (b) $E(Y/X = 3)$, and (c) $V(X/Y = 1)$.

Also examine whether the random variables X and Y are independent or not.

4 + 3 + 7 + 1 = 15

42. (a) Obtain the first four cumulants in terms of central moments.

(b) Show that for a random variable X having probability density function

$f(x) = \frac{\beta\alpha^{\beta}}{x^{\beta+1}}$ if $0 < \alpha \le x$, $\beta > 0$ and $f(x) = 0$, otherwise, the moment generating function does not exist for $\alpha > 0, \beta > 0$, but mean exists for $\beta > 1$.

(c) Define moment generating function in case of a bivariate distributions. **7 + 5 + 3 = 15**

43. Suppose that the time in minutes that a person has to wait at a certain bus stop for a bus is found to be a random phenomenon with probability distribution specified by the cumulative distribution function :

$$f(x)=\begin{cases}0, & x<0\\ \frac{x}{8}, & 0\le x<2\\ \frac{x^2}{16}, & 2\le x<4\\ 1, & x\ge 4\end{cases}$$

(a) Is the distribution function continuous? If so, find the probability density function.

(b) What is the probability that a person with have to wait (i) more than 2 minutes (ii) less than 2 minutes and (iii) between 1 and 2 minutes?

(c) What is the conditional probability that the person will have to wait for a bus for (i) more than 2 minutes given that it is more than one minute, and (ii) less than 2 minutes, given that it is more than 1 minute? **3 + 5 + 7 = 15**

44. (a) Find the moment generating function of sample mean of n independently and identically distributed random variables $\left(\overline{X}=\frac{X_1X_2+\ldots+X_n}{n}\right)$.

(b) The MGF of a random variable X is $M_X(t)=\frac{2}{5}+\frac{1}{3}e^{2t}+\frac{4}{15}e^{3t}$. Find the mean and variance of X.

(c) Point out the limitations of moment generating function.

(d) If X is a random variable with probability generating function $P(s)$, find the probability generating function of (i) $X+1$ and (ii) $2X$.

3 + 5 + 3 + 4 = 15

(2 × 15 = 30 Marks)