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Reg. No. : ....

Name : ....

Fifth Semester B.Sc. Degree Examination, December 2022

First Degree Programme under CBCSS

Statistics

Core Course VII

ST 1543 – TESTING OF HYPOTHESIS

(2018 Admission onwards)

Time : 3 Hours Max. Marks : 80

(Statistical Table and Calculator and permitted)

SECTION – A

Answer **all** questions. **Each** question carries **1** mark.

1. What is a simple hypothesis?

2. What do you mean by best critical region?

3. Define the power of a test.

4. What do you mean by most powerful test?

5. What are likelihood ratio tests?

6. What are the conditions to be satisfied to apply a Chi-square test for goodness of fit.

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7. Which test statistics is used to test for a hypothetical value of the proportion of observations when a sample is taken from a Binomial population?

8. What are the assumptions of a t-test?

9. Define the empirical distribution function.

10. What is the indication of the number of runs in a run test?

(10 × 1 = 10 Marks)

SECTION – B

Answer any **eight** questions. **Each** question carries **2** marks.

11. Distinguish between the level of the test α and the p value.

12. A population follows Normal distribution with parameters μ and σ = 3. To test the hypothesis $H_0 = \mu = 5$ vs $H_1 : \mu = 7$, the test procedure suggested is to reject H_0 if $\overline{x} \geq 6$ where $\overline{x}$ is the mean of a sample of size 16. Find the significance level and power of the test.

13. What are the steps for carrying out a statistical test procedure?

14. Using the Neymann-Pearson Lemma, find the best critical region for the test $H_0 : \mu = \mu_0$ vs $H_1 : \mu = \mu_1$, when μ_1 and σ^2 is known.

15. When do you call a test uniformly most powerful?

16. A coin is tossed 900 times in which head appears 490 times. Does it support the claim that the coin is unbiased?

17. Outline the large sample test for testing the specified variance of the population.

18. From a population with mean 200 and unknown SD, a sample is taken with mean 195 and S.D 50.If the null hypothesis is rejected at 5% level, what is the least sample size?

19. Discuss the applications of chi-square distribution.

20. Briefly discuss the test procedure of testing the equality means of the population with small samples.

21. The S.Ds of two samples of sizes 10 and 14 from two normal populations are 3.5 and 3.0 respectively. Examine whether the two populations have the same variances.

22. Define an F statistic. What are its uses?

23. In two Colleges affiliated to a University 40 out of 250 and 49 out of 200 students failed in an examination. If the overall percentage of failure in the University is 20, examine whether the two Colleges differ in performance significantly.

24. Discuss the runs test for randomness.

25. What is the procedure of a sign test for one sample?

26. Explain the Mann-Whitney U test. For which parametric test this is an alternative?

(8 × 2 = 16 Marks)

SECTION – C

Answer any **six** questions. **Each** question carries **4** marks.

27. Let $X \sim N(\mu, \sigma^2), \mu$ is the unknown mean and $\sigma^2 = 4$. To test $H_0 : \mu = -1$ vs $H_1 : \mu = 1$ based on a sample of size 10 from this population, we use the critical region $x_1 + 2x_2x + 3x_3 + \ldots + 10x_{10} \geq 0$. What is its size? What is the power of the test?

28. State the Neyman-Pearson lemma and give its relevance.

29. Examine whether a Best Critical Region exists for testing $H_0 : \theta = \theta_0$, vs $H_1 : \theta > \theta_0$ for the parameter θ of $f(x;\theta) = \frac{1+\theta}{(x+\theta)^2}, 1 \leq x < \infty$.

30. Mention the properties of likelihood ratio test.

31. A sample of 400 observations were taken from a population with S.D 15. If the mean of the sample is 27, test whether the hypothesis that the population mean is greater than 24.

32. A manufacturer of dry cells claimed that the life of their cells is 24.0 hours. A sample of 10 cells had mean life of 22.5 hours with a S.D 3.0 hours. On the basis of this information test whether the claim is valid or not.

33. Write the procedure for carrying out a chi-square test of homogeneity.

34. Test whether the following figures provide the evidence of the effectiveness on inoculation.

	Attacked	Not attacked
Inoculated	120	80
Not Inoculated	180	420

35. Two samples of 6 and 5 items respectively gave the following data: mean of the first sample =40,S.D of the first sample =8, mean of the second sample = 50 S.D of the second sample =10. Is the difference in means significant at 5% level?

36. What are the advantages and disadvantages of non-parametric methods over parametric methods?

37. A public school official felt that high school sections of a large school system would tend to score higher on a standard reading examination than the national median of 50. He randomly select 13 students and gave them the test. The results are as follows:57,70,42,48,72,63.45,66,59,39,73,78,47. Is his feeling correct? Use Wilcoxon's signed rank test. (From tables with n=13, $\alpha = .01$ table value C=13).

38. Write a comparison between Chi-square test of goodness-of—fit and a Kolmogrov-Smirnov test.

(6 × 4 = 24 Marks)

SECTION – D

Answer any **two** questions. **Each** question carries **15** marks.

39. (a) Show that every most powerful critical region is unbiased.

(b) Given a random sample of size n from a population with pdf $f(x,\sigma) = \frac{1}{\sigma} e^{-\frac{x}{\sigma}}, x > 0, \sigma > 0$. Show that there exists no UMP test for testing $H_0 : \sigma = \sigma_0, vs\, H_1 : \sigma \neq \sigma_0$.

40. (a) Give the procedure for testing the equality of means of two normal populations when the population variances are known.

(b) Two independent samples from two Normal populations are shown below. Test whether the two populations have the same variance.

Sample I	60	65	71	74	76	82	85	87		
Sample II	61	66	67	85	78	63	85	86	88	91

41. (a) A sample of 27 pairs of observations from a Normal population gives the sample correlation coefficient r = 0.6. Is it likely that the variables are correlated?

(b) Discuss the test for equality of correlation coefficients of two populations.

42. (a) Explain the paired t test, mentioning the assumptions involved.

(b) An IQ test was administered to 5 persons before and after they were trained. The results are as follows:

Candidate :	A	B	C	D	E
IQ before training :	110	120	123	132	125
IQ after training :	120	118	125	136	121

Test whether there is any change in IQ after the training.

43. (a) Discuss the test for equality of means of two normal populations (i) with known S,Ds and (ii) with the same but unknown S.D.

(b) Two samples from normal populations gave the following results.

Sample size	Mean	S.D.
12	1050	68
10	980	74

Do the samples come from the same population with $\sigma_1^2 = \sigma_2^2$, unknown.

44. (a) Explain the Kolmogrov-Smirnov one sample test procedure.

(b) Discuss the procedure of the Wilcoxon's matched pair signed rank test.

(2 × 15 = 30 Marks)

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Fifth Semester B.Sc. Degree Examination, December 2022

First Degree Programme under CBCSS

Statistics

Core Course V

ST 1541 – LIMIT THEOREMS AND SAMPLING DISTRIBUTIONS

(2018 Admission onwards)

Time : 3 Hours Max. Marks : 80

Statistical Table is permitted

SECTION – A

Answer **all** questions. **Each** question carries **1** mark.

1. Define sigma field.
2. If $A \subseteq B$ prove $P(A) \leq P(B)$, using axioms of Probability.
3. Define convergence in probability.
4. State Weak law of large numbers.
5. Define Chi square distribution with n degrees of freedom.
6. What is sampling distribution?
7. Define non central t distribution.
8. What is the relation between Student's t and F distribution?

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9. Define r[th] order statistic $X_{(r)}$.

10. If X_1, X_2, ... X_n be a random sample from $U(0,\theta)$, find the distribution function of $Max(X_1, X_2..., X_n)$.

(10 × 1 = 10 Marks)

SECTION – B

Answer any **eight** questions. **Each** question carries **2** marks

11. Define limit supremum and limit infimum for a sequence of events $\{A_n\}$.

12. Suppose $\{X_n\}$ is a sequence of random variables with probability mass function $P(X_n = 1) = \frac{1}{n}$ and $P(X_n = 0) = 1 - \frac{1}{n}$. Examine the convergence of $\{X_n\}$ in probability.

13. State Borel – Cantelli lemma.

14. What are the assumptions on Central Limit theorem?

15. A random variable X has mean 50 and variance 100. Use Chebychev's inequality to obtain the upper bound for $P\{|X - 50| \geq 15\}$.

16. List the advantages of Chebychev's inquality.

17. Let $X_1, X_2 ...., X_{100}$ be a random sample of size 100 drawn from a population with mean 10 and variance 9, then find $P(\overline{X} > 10.5)$ using central limit theorem.

18. Distinguish between parameter and statistic.

19. If the sample values are 1, 3, 5, 6, 9, find the standard error of the sample mean.

20. If $X_1, X_2, ..., X_n$ are independent $N(\mu, \sigma^2)$ random variables, find the distribution of $\sum_{i=1}^{n} \left(\frac{X_1 - \mu}{\sigma}\right)^2$.

21. Find the mode of the Chi-square distribution with n degrees of freedom.

22. What are the characteristics of *t* distribution?

23. What are the properties of non central F distribution?

24. Let X and Y be independent standard normal variates. State the distribution of $\frac{x^2}{y^2}$ and write down the pdf.

25. Let $(X_1, X_2 ... X_n)$ be a random sample of size n from a distribution with density function f(x) and distribution function F(x). Write down the distribution function and probability density function of the nth order statistic, $X_{(n)}$.

26. Le $(X_1, X_2 ... X_n)$ be a sequence of independent normal random variables with pdf $f(x) = e^{-(x-\theta)}, x > \theta$. Find the pdf of $Min(X_1, X_2 ... X_n)$.

(8 × 2 = 16 Marks)

SECTION – C

Answer any **six** questions. **Each** question carries **4** marks.

27. If $A_1, A_2, ..., A_n ...$ is a sequence of events in sample space S such that $A_1 \subseteq A_2 \subseteq ... \subseteq A_n ...$ then prove that $P\left(\bigcup_{n=1}^{\infty} A_n\right) = \lim_{n \to \infty} P(A_n)$.

28. Show by an example that a sequence of distribution functions need not always converge to a distribution function.

29. State and prove Bernoulli's law of large numbers.

30. Examine whether the weak law of large numbers holds good for the sequence $\{X_n\}$ of independent random variables where $P\left\{X_n = \frac{1}{\sqrt{n}}\right\} = \frac{2}{3}, P\{X_n = -\frac{1}{\sqrt{n}}\} = \frac{1}{3}$.

31. A random sample of size 16 is taken from a normal population with mean 30 and variance 64. Find the probability that the sample variance s^2 will be less than the population variance.

32. Give the properties of Chi square distribution and examine its relationship with the normal distribution.

33. What are the mean and variance of sample variance s^2?

34. If X_1, X_2, X_3 and X_4 are independent observations from a univariate normal with mean zero and unit variance. Find the distribution of (a) $U = \frac{\sqrt{2}\, x_3}{\sqrt{x_1^2 + x_2^2}}$ (b) $V = \frac{3x_4^2}{x_1^2 + x_2^2 + x_3^2}$.

35. If two independent random samples of sizes 15 and 20 are taken from $N(\mu, \sigma)$, what is $P\left(\frac{s_1^2}{s_2^2} < 2\right)$?

36. Give two examples of a statistic following Students *t* distribution.

37. Using Statistical table, find the right tailed critical value corresponding to area 0.05 for (a) Chi-square distribution with 15 degrees of freedom (b) *t* distribution with 16 degrees of freedom.

38. Let X1, X2, ... ,X_n be n independent variates, X_i having Geometric distribution with parameter $p_i, i.e., P(X_i = x_i) = q_i^{x_i-1} p_i, q_i = 1 - p_i, x_i = 1,2,3,...$ Show that $X_{(1)}$ is distributed geometrically with parameter $(1 - q_1 q_2 .... q_n)$.

(6 × 4 = 24 Marks)

SECTION – D

Answer any **two** questions. **Each** question carries **15** marks.

39. (a) State and prove Lindberg-Levy form of central limit theorem.

(b) Show by using central limit theorem that if X follows the binomial distribution with parameters (n, p), its distribution will tend to normal as $n \to \infty$.

40. (a) State and prove Chebychev's inequality.

(b) For the geometric distribution $f(x) = 2^{-x}, x = 1,2,3...$ prove that Chebychev's inequality gives $P\{|X - 2| \leq 2\} > 1/2$, while the actual probability is 15/16.

41. (a) Derive the sampling distribution of means of samples chosen from a normal population.

(b) A population is known to follow normal distribution with mean 2 and S.D. 3. Find the probability that the mean of a sample size 16 taken from this population will be greater than 2.5.

42. (a) Derive the moment generating function of Chi square distribution with n degrees of freedom and hence obtain its mean and variance.

(b) State and prove the additive property of Chi square distribution.

43. (a) Define t, χ^2 and F statistics and establish relationship between each of them.

(b) If F (m, n) represents a F variate with (m, n) degrees of freedom, prove that $P[F(m,n) \geq c] = \left[F(n,m) \leq \frac{1}{c}.\right]$

44. Let $X_1, X_2, \ldots, X_n$ be a random sample from a population with cumulative density. Show that $Y_1 = \text{Min}\ (X_1, X2, \ldots, X_{n,})$ is exponential with parameter $n\lambda$ if and only if each X_i is exponential with parameter λ.

(2 × 15 = 30 Marks)

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Fifth Semester B.Sc. Degree Examination, December 2022

First Degree Programme under CBCSS

Statistics

Core Course VI

ST 1542 – ESTIMATION

(2018 Admission onwards)

Time : 3 Hours Max. Marks : 80

SECTION A

Answer **all** questions. Each question carries **1** mark.

1. Define estimator.
2. Give an example of unbiased estimator.
3. Give the large sample property of a good estimator.
4. Define parametric space.
5. Give an example of a sufficient statistics.
6. Define minimum variance unbiased estimator.
7. Describe linear parametric function.
8. Define confidence coefficient.
9. Define likelihood function.
10. Define maximum likelihood estimator.

(10 × 1 = 10 Marks)

P.T.O.

SECTION B

Answer any **eight** questions. Each question carries **2** marks.

11. What do you understand by point estimation?

12. If T is an unbiased estimator of θ, examine whether T^2 is unbiased estimator of θ^2.

13. State Neymann's factorization theorem and mention its uses.

14. Show that in Cauchy distribution.

$$f(x, \theta) = \frac{1}{\pi\left[1+(x-\theta)^2\right]}, -\infty < x < \infty$$

Sample median is a consistent estimator of θ.

15. Find sufficient estimator of θ in $f(x, \theta) = \theta e^{-\theta x}, x > 0, \theta > 0$ if it exist.

16. What is sampling distribution?

17. Find the maximum likelihood estimator of λ in $P(\lambda)$.

18. Explain the method of construction of confidence interval.

19. Describe linear estimation.

20. Give any two important properties of moment estimator.

21. Find 95% confidence interval of population proportion.

22. Describe minimum variance bound estimator.

23. Describe the method of least squares.

24. Explain estimability.

25. Find the consistent estimator of population mean in normal population.

26. If $(x_1, x_2......x_n)$ is a random sample drawn from $N(\mu, 1)$ show that $t = 1/n \ \xi^n \ x^2$ is unbiased estimator of $\mu^2 + 1$.

(8 × 2 = 16 Marks)

SECTION C

Answer any **six** questions. Each question carries 4 marks.

27. State and prove the sufficient set of conditions for consistency of an estimator.

28. Let (X_1, X_2, X_3) be a random sample of size three drawn from a population with mean μ and variance σ^2. Let T_1, T_2 and T_3 be three estimators of μ. Where $T_1 = X_1 + X_2 - X_3$; $T_2 = 2X_1 + 3X_2 - 4X_3$; $T_3 = (X_1 + X_2 + X_3)/3$
    (a) Examine whether T_1, T_2 and T_3 are unbiased estimators of μ or not.
    (b) Which estimator is more efficient? Why?

29. Show that minimum variance unbiased estimator is unique.

30. Find a sufficient estimator of θ based on a random sample of size n drawn from uniform distribution with parameter θ.

31. A random sample $(x_1, x_2 ...... x_n)$ is taken from $N(0, \sigma^2)$. Examine whether $t = 1/n$ ξ^n x_i^2 is a minimum variance bound estimator of σ^2.

32. Explain Gauss-Markov set-up.

33. Obtain 100 (1 − α)% confidence interval of σ^2 based on sample observations drawn from $N(\mu, \sigma^2)$.

34. A random sample of 500 apples was taken from a large consignment and 60 were found to be bad apples. Find 98% confidence interval for the proportion of bad apples in the consignment.

35. Explain the desirable properties of a good estimator.

36. Examine whether sample variance is an unbiased and consistent estimator of population variance in $N(\mu, \sigma^2)$.

37. Obtain the moment estimator of θ based on sample observations drawn from
    $f(x, \theta) = \theta e^{-\theta x}$, $x > 0, \theta > 0$.

38. State and establish the necessary and sufficient condition for estimability of a linear parametric function.

(6 × 4 = 24 Marks)

SECTION D

Answer any **two** questions. Each question carries **15** marks.

39. (a) Discuss the method of maximum likelihood estimation. Also describe any two important properties of M.L.E.

(b) Obtain the maximum likelihood estimators of μ and σ^2 in $N(\mu, \sigma^2)$.

40. (a) Describe the method of moments. Examine whether moment estimators are

(i) unbiased

(ii) unique.

(b) Find the moment estimators of α and β in

$$f(x, \alpha, \beta) = \frac{\beta^2}{\alpha} e^{-\beta x} X^{\alpha-1}, 0 < X < \infty,$$

41. (a) Let $X_1, X_2, X_3, \ldots\ldots X_n$ be a random sample of size n drawn from a population with $f(x,\theta) = e^{-(x-\theta)}$, $0 < x < \infty$, $-\infty < \theta < \infty$

Find sufficient estimator of θ.

(b) Obtain the minimum variance bound estimator of μ in $N(\mu, \sigma^2)$ when σ^2 is known. Also find the variance of the estimator.

(c) Find an unbiased estimator of θ in $f(x,\theta) = \theta(1-\theta)^{x-1}$, $x = 1, \infty\ldots\ldots,$ $0 < \theta < 1$.

42. (a) Obtain the 100 $(1-\alpha)\%$ confidence interval of $P_1 - P_2$, where P_1 and P_2 are proportions of two independent populations.

(b) Show that for the distribution $f(x, \theta) = \theta e^{-\theta x}$, $x > 0$ the central confidence limits for large samples with 95% confidence coefficient are given by $\left(1 \pm \frac{1.96}{\sqrt{n}}\right) \Big/ \overline{x}$.

43. Find 100 $(1-\alpha)\%$ confidence interval of $\mu_1 - \mu_2$, where μ_1 and μ_2 are means of two independent normal populations when

(a) the population variances are unknown and different.

(b) when the population variances are common and unknown.

44. State and prove Gauss-Markov's theorem.

(2 × 15 = 30 Marks)
