(Pages : 3) **M – 3388**

Reg. No. : ....

Name : ....

Third Semester M.Sc. Degree Examination, December 2021

SDE

Mathematics

MM 231 : COMPLEX ANALYSIS — I

(2017 Admission Onwards)

Time : 3 Hours Max. Marks : 75

Answer either PART A or PART B of each question.

All questions carry equal marks.

1. (A) (a) Define absolutely convergent series. Prove that if $\sum a_n$ converges absolutely then $\sum a_n$ converges. **7**

(b) If G is open and connected and $f : G \to C$ is differentiable with $f'(z) = 0$ for all z in G, then prove that f is constant. **8**

OR

(B) (a) If $\gamma : [a, b] \to C$ is piecewise smooth then prove that γ is of bounded variation and $V(\gamma) = \int_a^b |\gamma'(t)| dt$. **10**

(b) If γ is piecewise smooth and $f : [a, b] \to C$ is continuous then $\int_a^b f d\gamma = \int_a^b f(t)\gamma'(t) dt$. **5**

P.T.O.

2. (A) (a) Define entire function. If f is an entire function then prove that f has a power series expansion with infinite radius of convergence. **5**

(b) Let G be a connected open set and let f be an analytic function defined on G. Prove that the following statements are equivalent.

(i) $f \equiv 0$

(ii) there is a point a in G such that $f^{(n)}(a) = 0$ for each $n \geq 0$.

(iii) $\{z \varepsilon G : f(z) = 0\}$ has a limit point in G. **10**

OR

(B) (a) State and Prove Fundamental Theorem of Algebra. **6**

(b) State and prove Maximum Modulus Theorem. **9**

3. (A) (a) State and prove Cauchy's integral formula 2nd version. **8**

(b) State and prove Independence of Path Theorem. **7**

OR

(B) State and prove Gaorsat's Theorem. **15**

4. (A) (a) If f has an isolated singularity at a then prove that $z = a$ is a removable singularity if and only if $\lim_{z \to a} (z-a) f(z) = 0$. **8**

(b) State and prove Residue Theorem. **7**

OR

(B) (a) State and Prove Casorati-Weierstrass Theorem. **7**

(b) Show that $\int_{-\infty}^{\infty} \frac{x^2}{1+x^4} dx = \frac{\pi}{\sqrt{2}}$. **8**

5. (A) (a) In the extended plane, find $d(z, z')$, the distance between z and z' and $d(z, \infty)$. **6**

(b) Obtain Schwarz's Lemma. **9**

OR

(B) (a) Let z_1, z_2, z_3, z_4 be four distinct points in C_∞. Then prove that (z_1, z_2, z_3, z_4) is a real number if and only if all four points lie on a circle. Also, prove that a Mobius transformation takes circles into circles. **10**

(b) State and prove Symmetry Principle. **5**

(5 × 15 = 75 Marks)

(Pages : 3) **M – 3391**

Reg. No. : ....

Name : ....

Third Semester M.Sc. Degree Examination, December 2021

SDE

Mathematics

MM 234 – GRAPH THEORY

(2017 Admission Onwards)

Time : 3 Hours Max. Marks : 75

Answer **five** questions choosing Part-A or Part-B from each question

All questions carry equal marks.

1\. (A) (a) Prove that two graphs are isomorphic if and only if their complements are isomorphic.

(b) Find the automorphism group of C_7.

(c) Let v be a vertex incident with a bridge in a connected graph G. Show that v is a cut-vertex of G if and only if $\deg v \geq 2$.

OR

(B) (a) Show that isomorphism is an equivalence relation on the set of all graphs.

(b) Prove that any two distinct blocks of a nontrivial connected graph have at most one vertex in common.

(c) Let G be a cubic graph. Prove that $\kappa(G) = \lambda(G)$.

P.T.O.

2\. (A) (a) Show that a nontrivial connected graph G is Eulerian if and only if every vertex of G has even degree.

(b) Prove that the Peterson graph is non-Hamiltonian.

OR

(B) (a) Obtain a necessary and sufficient condition for the Cartesian product $G \times H$ of two nontrivial connected graphs G and H to be Eulerian.

(b) Let G be a graph of order $n \geq 3$. If $\deg u + \deg v \geq n$ for each pair u, v of nonadjacent vertices of G, prove that G is Hamiltonian.

(c) Define Hamiltonian number and give an example.

3\. (A) (a) Show that every tournament contains a Hamiltonian path.

(b) State and prove the Marriage Theorem.

(c) Prove that the Peterson graph is the unique 5-cage.

OR

(B) (a) Let G be a bipartite graph with partite sets U and W such that $r = |U| \leq |W|$. Prove that G contains a matching of cardinality r if and only if U is neighborly.

(b) For every integer $k \geq 1$, prove that the complete graph K_{2k+1} is Hamiltonian factorable.

4\. (A) (a) Describe the problem of the Five princes and the problem of the Five Places.

(b) Let k and n be integers with $2 \leq k < n$. Among all graphs of order n that do not contain K_{k+1} as a subgraph, prove that at least one of those having maximum size is a k-partite graph.

(c) Find $RR(F, P_3)$.

OR

(B) (a) Show that a graph G is 2-chromatic if and only if G is bipartite.

(b) Define edge chromatic number of a graph G. Give an example of a graph G which is 4edge chromatic.

(c) For every tree T_m of order $m \geq 2$ and every integer $n \geq 2$, prove that $r(T_m, K_n) = (m-1)(n-1)+1$.

5. (A) (a) Define radius and diameter of a graph G. Prove that $rad(G) \leq \dim(G) \leq 2\, rad\,(G)$.

(b) Define locating number of a graph G. Prove that a connected graph G of order n has locating number 1 if and only if $G \cong P_n$.

(c) For each pair a, b of positive integers with $a \leq b \leq 2a$, prove that there exists a connected graph G with $rad_D(G) = a$ and $diam_D(G) = b$.

OR

(B) (a) Prove that the center of every connected graph G is a subgraph of some block of G.

(b) Let G be a connected graph. Prove that a vertex v is a boundary vertex of G if and only if v is not an interior vertex of G.

(c) Define radio k-coloring. Give an example of a graph G for which $rc_2(G) = 6$.

(5 × 15 = 75 Marks)

Name : ....

Third Semester M.Sc. Degree Examination, November 2022

SDE

Mathematics

MM 232 : FUNCTIONAL ANALYSIS I

(2017 Admission Onwards)

Time : 3 Hours Max. Marks : 75

(1) Answer either Part A or Part B of each questions.

(2) All question carry equal marks.

1. (A) (a) Prove that the sequence space l^p, $1 \leq p < \infty$ is a normed linear space. **5**

(b) If Y is a closed subspace of a normed space X, define the quotient norm on X/Y. Verify that it is a norm on X/Y. Prove that a sequence $\{X_n + y\}$ in X/Y converges to $X + Y$ in X/Y if and only if there is a sequence (Y_n) in Y such that $(X_n + Y_n)$ converges to X in X. **10**

OR

(B) (a) If Y is a finite dimensional subspace of a normed space X, prove that Y is complete. **6**

(b) Let X denote a subspace of B(T) with sup norm, $1 \in X$ and f be a linear functional on X. If f is continuous and $\|f\| = f(1)$, then prove that f is positive. Further if $\operatorname{Re} x \in X$ whenever $x \in X$ and if f is positive, then prove that f is continuous and $\|f\| = f(1)$. **9**

P.T.O.

$g \in Y^1$, there is a unique Hahn-Banach extension of g to X if and only if X^1 is strictly convex. **10**

OR

(B) (a) Show that a normed space X is a Banach space if and only if every absolutely summable series of elements in X is summable in X. **7**

(b) If Y is a closed subspace of a normed space, show that X is a Banach space if and only if Y and X/Y are Banach spaces in the induced norm and the quotient norm, respectively. **8**

3. (A) (a) State and prove Uniform boundedness principle. **9**

(b) State and prove Banach-Steinhaus theorem. **6**

OR

(B) (a) State and prove the closed graph theorem. **9**

(b) If X and Y are normed spaces and $F : X \to Y$ is linear, show that F is an open map if and only if there exists some $\gamma > 0$ such that for every $y \in Y$, there is some $x \in X$ with $F(x) = y$ and $\|x\| \le \gamma\|Y\|$. **6**

4. (A) (a) If X is a normed space and $A \in BL(X)$ is of finite rank, prove that

$$\sigma_e(A) = \sigma_a(A) = \sigma(A).$$ **9**

(b) If X is a Banach space, $A \in BL(X)$ and $\|A^p\| < 1$ for some positive integer p, prove that the bounded operator I-A is invertible. **6**

OR

(B) If X is a nonzero Banach space over $\mathbb{C}$ and $A \in BL(X)$, prove that

(a) $\sigma(A)$ is non empty **7**

(b) $r_\sigma(A) = \lim_{n \to \infty} \|A^n\|^{1/n}$ **8**

(b) X^1 is reflexive. 5

(c) Every closed subspace of X is reflexive. 6

OR

(B) (a) Prove that the normed space l^∞ is not reflexive. 8

(b) If X is a normed space and $A \in CL(X)$, $\sigma_a(A) = \sigma(A)$ 7

(5 × 15 = 75 Marks)