(Pages : 4) **P – 7696**

Reg. No. : ....

Name : ....

First Semester B.Sc. Degree Examination, March 2023

First Degree Programme under CBCSS

Mathematics

Core Course

MM 1141 : METHODS OF MATHEMATICS

(2018 Admission Onwards)

Time : 3 Hours Max. Marks : 80

SECTION – I

All questions are compulsory. Each question carries **1** mark.

1. What is the local linear approximation of $f(x)=\sqrt{x}$ at $x_0 = 1$.

2. Define point of inflection.

3. Define critical point.

4. State Extreme value theorem.

5. For a particle in rectilinear motion, the acceleration and position functions $a(t)$ and $s(t)$ are related by the equation ————

6. Let $A(x)$ be the area under the graph of a nonnegative continuous function f over an interval $[a, x]$, then $A'(x) =$ ————.

7. Integrals over infinite intervals are known as ————

P.T.O.

8. $\cosh x + \sinh x =$ ________.

9. Define the work done by a force F.

10. The total mass of a homogeneous lamina of area A and density δ is ________.

(10 × 1 = 10 Marks)

SECTION – II

Answer **any eight** questions. Each question carries **2** marks.

11. Evaluate $\lim_{x \to 0^+} \left(\frac{1}{x} - \frac{1}{\sin x} \right)$.

12. Find the subintervals of $[0, 2\pi]$ in which the function $f(x) = x + 2\sin x$ is decreasing.

13. Find all critical points of $f(x) = x^3 - 3x + 1$.

14. What are the geometrical implications of the multiplicity of a root of a polynomial?

15. Find the horizontal and vertical asymptotes of the curve given by $y = \frac{\ln x}{x}$.

16. Find the absolute extrema of $f(x) = 6x^{4/3} - 3x^{1/3}$ on the interval $[-1, 1]$.

17. Suppose that a particle moves on a coordinate line so that its velocity at time t is $v(t) = t^2 - 2t\ m/s$. Find the distance traveled by the particle during the time interval $0 \le t \le 3$.

18. Find the average value of the function $f(x) = \sqrt{x}$ over the interval [1, 4].

19. Define hyperbolic sine and draw its graph.

20. Define improper integral. Is $\int_0^3 \frac{dx}{x^2 - 3x + 2}$ an improper integral? Explain.

21. Use Pappus Theorem to find the volume V of the torus generated by revolving a circular region of radius b about a line at a distance a (greater than b) from the rater of the circle.

22. Evaluate $\int_0^\infty e^{-x} dx$.

(8 × 2 = 16 Marks)

SECTION – III

Answer **any six** questions. Each question carries **4** marks.

23. Evaluate $\lim_{x \to 0} (\cos x)^{1/x^2}$.

24. Find all the inflection points of $f(x) = xe^{-x}$.

25. Find the radius and height of the right circular cylinder of largest volume that can be inscribed in a right circular cone with, radius 6 inches and height 10 inches.

26. State and prove Rolle's theorem.

27. Find the volume of the solid generated when the region between the graphs of the equations $f(x) = \frac{1}{2} + x^2$ and $g(x) = x$ over the interval [0, 2] is revolved about the x-axis.

28. Using the notion of surface of revolution, show that the area of the surface of a sphere of radius r is $4\pi r^2$.

29. Find the length of the arc of the curve $y^2 = x^3 3$ from the origin to the point (1, 1).

30. A spring exerts a force of 5 N when stretched 1 m beyond its natural length.

    (a) Find the spring constant k.

    (b) How much work is required to stretch the spring 1.8 m beyond its natural length?

31. Evaluate $\int_0^\infty (1 - x)e^{-x} dx$.

(6 × 4 = 24 Marks)

SECTION – IV

Answer **any two** questions. Each question carries **15** marks.

32. (a) Find the radius and height of the right circular cylinder of largest volume that can be inscribed in a right circular cone with radius 6 inches and height 10 inches. **5**

(b) Using Roll's theorem show that between any two real root of $e^{-x} = \sin x$, there is at least one real root of $e^{-x} = -\cos x$. **5**

(c) Find the points of inflection of the cubic $y = \frac{a^2 x}{x^2 + a^2}$. **5**

33. (a) Explain the 7 steps in sketching the graph of a rational function. **6**

(b) Sketch the graph of $y = \frac{x^2 - 1}{x^3}$. **9**

34. (a) Find the length of the curve $y = \log \sec x$ between the points given by $x = 0$ and $x = \pi/3$. **5**

(b) Find the volume when the loop of the curve $y^2 = x(2x-1)^2$ revolves about the *x*-axis. **5**

(c) Find the area of the surface that is generated by revolving the portion of the curve $y = x^2$ between $x = 1$ and $x = 2$ about the y-axis. **5**

35. (a) A space probe of mass $m = 5.00 \times 104$ kg travels in deep space subjected only to the force its own engine. Starting at a time when the speed of the probe is v = 1.10 x 104 m/s. the engine is fired continuously over a distance of 2.50 × 106 m with a constant force of 400 × 105 N in the direction of motion. What is the final speed of the probe? **6**

(b) Evaluate $\int_1^4 \frac{dx}{(x-2)^{2/3}}$. **5**

(c) Find the mass and center of gravity of the lamina bounded by the x-axis, the line $x = 1$, and the curve $y = \sqrt{x}$. Given $\delta = 2$. **4**

(2 × 15 = 30 Marks)

(Pages : 4)

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Reg. No. : ....

Name : ....

First Semester B.Sc. Degree Examination, March 2023

First Degree Programme under CBCSS

Mathematics

Complementary Course I for Statistics

MM 1131.4 : MATHEMATICS I – BASIC CALCULUS FOR STATISTICS

(2018 - 2020 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the first ten questions are compulsory. Each question carries 1 mark.

1. Compute the derivative of $f(x) = \dfrac{2x^2+4x+3}{x^2+2x+1}$

2. Find $\dfrac{dy}{dx}$ if $5y^2 + \sin y = x^2$

3. State Rolle's theorem.

4. Evaluate the sum $\sum_{n=0}^{N} \ln \dfrac{n+1}{n}$.

5. Sum the even numbers between 2000 and 3000 inclusive.

6. Write the Maclaurin series for $\dfrac{1}{1+x}$.

P.T.O.

7. What is the radius of convergence of the complex power series $P(z)=\sum_{n=0}^{\infty}(-1)^n\left(\frac{z}{2}\right)^n$.

8. Evaluate the integral $\int_1^{\infty}\frac{dx}{x^3}$.

9. Evaluate the integral $\int \ln x dx$.

10. Find the mean value of the function $f(x)=x^3$ between the limits $x=0$ and $x=2$.

(10 × 1 = 10 Marks)

SECTION – II

Answer any **eight** questions. Each question carries **2** marks.

11. Using Leibnitz' theorem find the third derivative of the function $f(x)=x^4\cos x$.

12. Find the lowest value taken by the function $f(x)=x^3-3x^3+4$.

13. Determine inequalities satisfied by $\ln x$: for suitable range of the real variable x.

14. Find an interval $[a, b]$ on which $f(x)=x^4+x^3-x^2+x-2$ satisfies the hypothesis of Rolle's theorem.

15. Consider a ball that drops from a height of 27m and on each bounce retains only a third of its kinetic energy; thus after one bounce it will return to a height of 9m, after two bounces to 3m, and so on. Find the total distance travelled between the first bounce and the Mth bounce.

16. Sum the series $S=1+\frac{4}{5}+\frac{7}{5^2}+\frac{10}{5^3}+\ldots$

17. Use the preliminary test to decide whether the series $\sum_{n=1}^{\infty}\frac{n!}{n!+1}$ is divergent or require further testing.

18. Use D-Alembert's ratio test to find whether the series $\sum_{n=1}^{\infty}\frac{2^n}{n^2}$ converges or diverges.

19. Sum the series $\sum_{n=1}^{N}(n+1)(n+3)$.

20. Evaluate $\int x^3 e^{x^2} dx$.

21. Evaluate the integral $\int e^{3x} \cos 2x \, dx$.

22. Evaluate the integral $\int_{-\infty}^{0} \frac{dx}{(2x-1)^3}$.

(8× 2= 16 Marks)

SECTION – III

Answer any **six** questions. Each question carries **4** marks.

23. Find the radius of curvature of the curve $x^{2/3} + y^{2/3} = a^{2/3}$ in the first quadrant.

24. Use the difference method to sum the series $\sum_{n=1}^{N} \frac{1}{n(n+1)(n+2)}$.

25. Find the first few terms of the Taylor series for $\ln x$ about $x = 1$.

26. Given that $\sum_{n=2}^{\infty} \frac{3^n}{n^5}$ diverges. Determine whether the series $\sum_{n=2}^{\infty} \frac{3^n - n^3}{n^5 - 5n^2}$ converges.

27. Find the real values of x for which the series $\sum_{n=1}^{\infty} \frac{x^n}{n+1}$ is convergent.

28. Evaluate the integral $\int_{1}^{4} \frac{dx}{(x-2)^{2/3}}$

29. Find the length of the curve $y = \sqrt{4 - x^2}$ over the interval $[0, 2]$.

30. Show that the value of the integral $I = \int_{0}^{1} \frac{1}{(1 + x^2 + x^3)^{1/2}} dx$ lies between 0.810 and 0.882.

31. The circle $x^2 + y^2 = a^2$ is rotated about the x - axis to generate a solid. Find its volume.

(6 × 4 = 24 Marks)

SECTION – IV

Answer any **two** questions. **Each** question carries **15** marks.

32. (a) Sum the series $S(x) = \frac{x^4}{3(0!)} + \frac{x^5}{4(1!)} + \frac{x^6}{5(2!)} + \ldots$

(b) Find the Maclaurin series for $\exp(\sin x)$.

33. (a) Sum the series $S(\theta) = 1 + \cos\theta + \frac{\cos 2\theta}{2!} + \frac{\cos 3\theta}{3!} + \ldots$

(b) Prove that $\cos\theta + \cos(\theta+\alpha) + \ldots + \cos(\theta+n\alpha) = \frac{\sin\frac{1}{2}(n+1)\alpha}{\sin\frac{1}{2}\alpha}\cos\left(\theta+\frac{1}{2}n\alpha\right)$.

34. Use integration by parts to find a relation between I_n and I_{n-1} where $I_n = \int_0^1 (1-x^3)^n\, dx$ and n is any positive integer. Hence evaluate $I_2 = \int_0^1 (1-x^3)^2\, dx$.

35. Suppose that the region between the x - axis and the line $y = 2x$ is revolved about the x - axis.

(a) Find the volume of the solid that is generated between $x = 0$ and $x = 5$

(b) Find the surface area of the above solid.

(2 × 15 = 30 Marks)

(Pages : 4) **P – 7760**

Reg. No. : ....

Name : ....

First Semester B.Sc. Degree Examination, March 2023

First Degree Programme under CBCSS

Mathematics

Complementary Course I for Statistics

MM 1131.4 : MATHEMATICS I – BASIC CALCULUS FOR STATISTICS

(2018 - 2020 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the first **ten** questions are compulsory. Each question carries **1** mark.

1. Compute the derivative of $f(x)=\dfrac{2x^2+4x+3}{x^2+2x+1}$

2. Find $\dfrac{dy}{dx}$ if $5y^2+\sin y=x^2$

3. State Rolle's theorem.

4. Evaluate the sum $\sum_{n=0}^{N}\ln\dfrac{n+1}{n}$.

5. Sum the even numbers between 2000 and 3000 inclusive.

6. Write the Maclaurin series for $\dfrac{1}{1+x}$.

P.T.O.

7. What is the radius of convergence of the complex power series $P(z) = \sum_{n=0}^{\infty} (-1)^n \left(\frac{z}{2}\right)^n$.

8. Evaluate the integral $\int_1^{\infty} \frac{dx}{x^3}$.

9. Evaluate the integral $\int \ln x dx$.

10. Find the mean value of the function $f(x) = x^3$ between the limits $x = 0$ and $x = 2$.

(10 × 1 = 10 Marks)

SECTION – II

Answer any **eight** questions. Each question carries **2** marks.

11. Using Leibnitz' theorem find the third derivative of the function $f(x) = x^4 \cos x$.
12. Find the lowest value taken by the function $f(x) = x^3 - 3x^3 + 4$.
13. Determine inequalities satisfied by $\ln x$ for suitable range of the real variable x.
14. Find an interval $[a, b]$ on which $f(x) = x^4 + x^3 - x^2 + x - 2$ satisfies the hypothesis of Rolle's theorem.
15. Consider a ball that drops from a height of 27m and on each bounce retains only a third of its kinetic energy; thus after one bounce it will return to a height of 9m, after two bounces to 3m, and so on. Find the total distance travelled between the first bounce and the M[th] bounce.
16. Sum the series $S = 1 + \frac{4}{5} + \frac{7}{5^2} + \frac{10}{5^3} + \ldots$
17. Use the preliminary test to decide whether the series $\sum_{n=1}^{\infty} \frac{n!}{n!+1}$ is divergent or require further testing.
18. Use D-Alembert's ratio test to find whether the series $\sum_{n=1}^{\infty} \frac{2^n}{n^2}$ converges or diverges.

19. Sum the series $\sum_{n=1}^{N}(n+1)(n+3)$.

20. Evaluate $\int x^3 e^{x^2} dx$.

21. Evaluate the integral $\int e^{3x}\cos 2x\, dx$.

22. Evaluate the integral $\int_{-\infty}^{0}\frac{dx}{(2x-1)^3}$.

(8× 2= 16 Marks)

SECTION – III

Answer any **six** questions. Each question carries **4** marks.

23. Find the radius of curvature of the curve $x^{2/3}+y^{2/3}=a^{2/3}$ in the first quadrant.

24. Use the difference method to sum the series $\sum_{n=1}^{N}\frac{1}{n(n+1)(n+2)}$.

25. Find the first few terms of the Taylor series for $\ln x$ about $x=1$.

26. Given that $\sum_{n=2}^{\infty}\frac{3^n}{n^5}$ diverges. Determine whether the series $\sum_{n=2}^{\infty}\frac{3^n-n^3}{n^5-5n^2}$ converges.

27. Find the real values of x for which the series $\sum_{n=1}^{\infty}\frac{x^n}{n+1}$ is convergent.

28. Evaluate the integral $\int_{1}^{4}\frac{dx}{(x-2)^{2/3}}$.

29. Find the length of the curve $y=\sqrt{4-x^2}$ over the interval $[0,2]$.

30. Show that the value of the integral $I=\int_{0}^{1}\frac{1}{\left(1+x^2+x^3\right)^{1/2}}dx$ lies between 0.810 and 0.882.

31. The circle $x^2+y^2=a^2$ is rotated about the x - axis to generate a solid. Find its volume.

(6 × 4 = 24 Marks)

SECTION – IV

Answer any **two** questions. **Each** question carries **15** marks.

32. (a) Sum the series $S(x)=\frac{x^4}{3(0!)}+\frac{x^5}{4(1!)}+\frac{x^6}{5(2!)}+....$

(b) Find the Maclaurin series for $\exp(\sin x)$.

33. (a) Sum the series $S(\theta)=1+\cos\theta+\frac{\cos 2\theta}{2!}+\frac{\cos 3\theta}{3!}+...$

(b) Prove that $\cos\theta+\cos(\theta+\alpha)+...+\cos(\theta+n\alpha)=\frac{\sin\frac{1}{2}(n+1)\alpha}{\sin\frac{1}{2}\alpha}\cos\left(\theta+\frac{1}{2}n\alpha\right)$.

34. Use integration by parts to find a relation between I_n and I_{n-1} where $I_n=\int_0^1(1-x^3)^n\,dx$ and n is any positive integer. Hence evaluate $I_2=\int_0^1(1-x^3)^2\,dx$.

35. Suppose that the region between the x - axis and the line $y=2x$ is revolved about the x - axis.

(a) Find the volume of the solid that is generated between $x=0$ and $x=5$

(b) Find the surface area of the above solid.

(2 × 15 = 30 Marks)

(Pages : 4)

P – 7685

Reg. No. : ....

Name : ....

First Semester B.A. Degree Examination, March 2023

First Degree Programme Under CBCSS

Mathematics

Complementary Course I for Economics

MM 1131.5 : MATHEMATICS FOR ECONOMICS I

(2013-2020 Admission)

Time : 3 Hours

Max. Marks : 80

SECTION – I

All the first ten questions are compulsory. They **carry 1** mark.

1. Express $\sin(x^3)$ as a composition of two functions.

2. Evaluate $\lim_{x\to 1}\frac{x^5-1}{x-1}$

3. Find $\frac{dy}{dx}$ where $y = x^2(3x^7-1)$

4. For a linear cost function TC = a + b q, the marginal cost is a constant.

5. Find the slope of the tangent line to the graph of $x+y+xy=3$ at (1,1).

6. Give an example of a function that is continuous but not differential at a point.

7. Obtain a relation between x and y if $x=t$ and $y=\sqrt{t}$

8. If $f(x)=x^n$ and $f'(1)=10$, find the value of n.

P.T.O.

9. Write down the derivative of $\log_a x$.

10. What is the value of $\lim_{x \to 0} \frac{\sin x}{x}$

(10 × 1 = 10 Marks)

SECTION – II

Answer **any eight** questions. These questions carry **2** marks each.

11. Find the natural domain of the function $f(x) = \sqrt{x^2 - 5x + 6}$

12. Draw the graph of $y = x^2$

13. If the total cost function is it $\pi = 2x^2 - 5x + 8$, find out the marginal cost when $x = 20$.

14. For the demand function $q = 25 - 4p + p^2$, show that the elasticity of demand for p = 5 is unitary.

15. If $y = x^4 - 4x^3 + 6x^2 - 4x - 3$, show that $\frac{dy}{dx} = 4(x-1)^3$.

16. Find $\frac{dy}{dx}$ if $y = \frac{1}{\sqrt{x^2 + 1}}$

17. Suppose that f and g are continuous functions such that $\lim_{x \to 2}[f(x) + 4g(x)] = 13$ and $f(2) = 1$. Find (a) g(2) (b) $\lim_{x \to 2 g(x)}$

18. For what *values* of x is there a discontinuity in the graph of $y = \frac{x^2 - 9}{x^2 - 5x + 6}$

19. If the demand law is given by $p = \frac{a}{x} - c$, show that the total revenue decreases as output increases.

20. Differentiate the following with respect to x: $\frac{x^2 - 1}{x^2 + 1}$

21. Find the slope of the tangent to the curve $y = ax + b + c/x$ at the point with abscissa x_1.

22. If x and y satisfy the relation $x^2 + y^2 = 4$, show that $\frac{dy}{dx} = \frac{-x}{y}$.

(8 × 2 = 16 Marks)

SECTION – III

Answer any **six** questions. These question carry **4** marks each.

23. Find the value of k so that

$$f(x) = \begin{cases} \frac{x^2 - 16}{x - 4}, & x \neq 4 \\ k, & x = 4 \end{cases}$$ is a continuous function.

24. Show that $f(x) = \begin{cases} x^2 + 1, & x \leq 1 \\ 2x, & x > 1 \end{cases}$ is continuous at $x = 1$. Sketch the graph of *f*.

25. At what points, if any does the graph of $y = x^3 - 3x + 4$ have a horizontal tangent line?

26. If $f(x)$ is a single valued function of x, express the derivatives of $\sqrt{f(x)}$ and its reciprocal in terms of the derivative of $f(x)$.

27. Find $\frac{dy}{dx}$ when (a) $xy + y^2 = 4$ (b) $x = t + 1;\ y = t^2 + 1$

28. Show that $f(x) = x^2$ has no inverse.

29. Find the function inverse to $y = \frac{2x + 1}{x - 1}$ and show that it is single valued. Represent it graphically.

30. If $\lim_{x \to +\infty} f(x) = 2$ and $\lim_{x \to +\infty} g(x) = -3$, find whether the limit $\lim_{x \to +\infty} \frac{2f(x) + 3g(x)}{3f(x) + 2g(x)}$ exists or not.

31. If $\lim_{x \to 2} \frac{x^n - 2^n}{x - 2} = 80$ and *n* is a positive integer, find the value of *n*

(6 × 4 = 24 Marks)

SECTION – IV

Answer **any two** questions. **These** question carries **15** marks each.

32. (a) If $f(x) = x^2 + 3$ and $g(x) = \sqrt{x}$, find $(gof)(x)$ and $(fog)(x)$

(b) Find the domain and range of $f(x) = \dfrac{1}{\sqrt{(x-1)(3-x)}}$.

(c) If $f(x) = x^2 - 3x + 1$, for what values x is $2f(x) = f(2x)$?

33. (a) Given that the function $f(x) = \begin{cases} 5-x, & x \neq 4 \\ 0, & x = 4 \end{cases}$

(i) Draw the graph of the function

(ii) Identify the discontinuity of the function in the graph

(iii) Find $\lim_{x\to 4}(5-x)$ and show that the value of the value of the limit is not equal to the value of the function at $x = 4$. What do you conclude?

(b) From the function $xy + 2x + y - 1 = 0$, find the limit of y as $x \to 1$, and the limit of x as $y \to 1$

34. (a) Explain the total revenue curve, average and marginal revenue curves.

(b) The total revenue received from sale of x units of a product is given by $R(x) = 12x + 2x^2 + 6$. Find (i) the average revenue (ii) the marginal revenue (iii) marginal revenue at x = 50 (iv) the actual revenue from selling 51^{st} item.

35. Find $F'(1)$ given that $f(1) = -1$, $f'(1) = 2$, $g(1) = 3$, and $g'(1) = -1$

(a) $F(x) = 2f(x) - 3g(x)$

(b) $F(x) = [f(x)]^2$

(c) $F(x) = f(x)g(x)$

(d) $F(x) = f(x)/g(x)$

(2 × 15 = 30 Marks)

(Pages : 4) **P – 7686**

Reg. No. : ....

Name : ....

First Semester B.A. Degree Examination, March 2023

First Degree Programme under CBCSS

Mathematics

Complementary Course for Economics

MM 1131.5 : MATHEMATICS FOR ECONOMICS I

(2021 Admission Onwards)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the first **ten** questions are compulsory They carry **1** mark each.

1. Give an example of a set with three elements.

2. State De Morgan's law.

3. Define union of two sets A and B.

4. How many subsets are there for {a,b}.

5. Every subset of an infinite set is infinite. True or false?

6. What is the degree of $x^2 - 2x + 3 = 0$.

7. Solve $3x - 6 = 0$.

P.T.O.

8. Write the general form of a quadratic equation.

9. Solve $5x = 8$.

10. Write the condition for a quadratic equation to have real solution.

(10 × 1 = 10 Marks)

SECTION – II

Answer any **eight** questions. These questions carry **2** marks each.

11. Write all subsets of {a,b}.

12. If $A = \{1,2,3,4\}$ and $B = \{a,2,b,4\}$. Find $A - B$.

13. If $A = \{a,b\}$ and $B = \{1,2,3\}$. Find A × B.

14. If $A = \{a,b,c\}$ and $B = (c,d,e\}$. Find A U B.

15. Define a relation on a set A.

16. If $A = \{1,2,3,4\}$ and $B = \{a,2,b,4\}$. Find $A \cap B$.

17. Solve $x^2 - 3x + 2 = 0$.

18. Solve $5x + 3 = 2x + 9$.

19. Solve $x + y = 4, x - y = 2$.

20. Given Supply: $q_s = 5P + 10$ and Demand: $q_d = -3P + 50$, find equilibrium price.

21. Solve $\frac{a}{x} = \frac{b}{c}$.

22. Solve $x^2 - 15x + 36 = 0$.

(8 × 2 = 16 Marks)

SECTION – III

Answer any **six** questions. These questions carry **4** marks each.

23. Find the power set of $A = \{1,2,3\}$.

24. Find $A \times B$, where $A = \{x \mid x = 1,2\}$ and $B = \{y \mid y = x - 1\}$.

25. Find the domain and range of the relation $R = \{(x,y) \in A \times B \mid x = 2y\}$, where $A = \{1,2,3,4,5,6\}$ and $B = \{1,2,3,4,5,6,7,8,9,10\}$.

26. Let $A = B = \{1,2,3,4\}$ and let R be the relation xRy if $x < y$. Find xRy.

27. Prove with the help of an example that $A \times B \neq B \times A$.

28. Solve $x^2 + 10x + 21 = 0$.

29. Solve $x - 2y = 0, 3x + 4y = 20$.

30. Solve $x^2 + 8x - 9 = 0$.

31. A market demand curve is given by $D = 50 - 5p$. Find the maximum price any body will pay for a commodity.

(6 × 4 = 24 Marks)

SECTION – IV

Answer any **two** questions. These questions carry **15** marks each.

32. (a) If $A = \{1,2,3\}$ and $B = \{4,5\}$. Find $A \times B, B \times A, A \times A$ and $B \times B$.

    (b) Let $A = \{1,2,3\}$ and $B = (4,5,6\}$. Find xRy if R is given by $y = x + 3$.

33. Let $U = \{1,2,3,4,5,6,7,8,9,10\}$, $A = \{1,2,3,4,5\}$, $B = \{2,4,6,8\}$ and $C = \{6,8,10\}$. Find $A - B$, $B - C$, A' and $A U B U C$.

34. Solve the following quadratic equations:

(a) $x^2 - 10x + 7 = 0$

(b) $x^2 - 8x + 16 = 0$

(c) $x^2 - 7x - 30 = 0$

35. Solve the following equations.

(a) $4x - 3y = 1, 2x + 9y = 4$.

(b) $2x + 3y = 3, 6x + 6y = -1$.

(2 × 15 = 30 Marks)

(Pages : 4)

P – 7711

Reg. No. : ....

Name : ....

First Semester B.Sc. Degree Examination, March 2023

First Degree Programme under CBCSS

Mathematics

Complementary Course for Physics

MM 1131.1 : MATHEMATICS – I – CALCULUS WITH APPLICATIONS IN PHYSICS – I

(2018-2020 Admission)

Time : 3 Hours

Max. Marks : 80

PART – I

Answer **all** questions. Each question carries **1** mark.

1. Find the derivative of $f(x) = \frac{\sin x}{x}$ with respect to x.

2. Stationary point of inflection a function $f(x)$ is a point at which ____________

3. State Rolle's theorem.

4. The area of a circle with radius a is ____________

5. Define the mean value of a function.

6. Define absolute convergence.

7. State D'Alembert's ratio test.

P.T.O.

8. If $v = -2i + k$, $w = 3i + 5j - 4k$ then find $v + w$.

9. Find the magnitude of the vector $2i + 3j + 6k$.

10. If the vectors a, b and c are coplanar, then $a \cdot (b \times c) = ...$

(10 × 1 = 10 Marks)

PART – II

Answer any **eight** questions. Each question carries **2** marks.

11. Find the derivative with respect to x of $f(t) = 2at$, where $x = at^2$.

12. Find the derivative with respect to x of $f(x) = x^3 \sin x$.

13. Evaluate $\int x \sin x \, dx$.

14. Evaluate the integral $I = \int_0^\infty \frac{x}{(x^2 + a^2)^2} dx$.

15. Find the volume of a cone enclosed by the surface formed by rotating about the x-axis the line $y = 2x$ between $x = 0$ and $x = h$.

16. Use Leibnitz' theorem to find the third derivative of the function $x^3 \sin x$.

17. Find Sum the integers between 1 and 200 inclusive.

18. Define interval of convergence of a power series and find the interval of convergence of the power Series $\sum_{k=0}^{\infty} \frac{x^k}{k!}$.

19. Find the Maclaurin series for $f(x) = \cos x$.

20. Determine whether the following series $\sum_{n=1}^{\infty} \left(\frac{1}{n}\right)^n$ converges.

21. Two particles have velocities $v_1 = i + 3j + 6k$ and $v_2 = i + 3j - 2k$, respectively. Find the velocity of the second particle relative to the first.

22. Find the area of the parallelogram with sides $a = i + 2j + 3k$ and $b = 4i + 5j + 6k$.

(8 × 2 = 16 Marks)

PART – III

Answer any **six** questions. Each question carries **4** marks.

23. Using logarithmic differentiation, find the derivative of $y = x^x$.

24. Find the positions and natures of the stationary points of the function $f(x) = 2x^3 - 3x^2 - 36x + 2$.

25. Find the radius of curvature of $x^2 + y^2 = 1$.

26. Evaluate the integral $\int \frac{2}{1+3\cos x} dx$.

27. Find the length of the curve $y = x^{3/2}$ from $x = 0$ to $x = 2$.

28. Evaluate the sum $\sum_{n=1}^{N} \frac{1}{n(n+1)}$.

29. Determine whether the series $\sum_{n=1}^{\infty} (-1)^{n+1} \frac{1}{n}$ converges.

30. Find the angle between the vectors $a = i + 2j + 3k$ and $b = 2i + 3j + 4k$.

31. Find the minimum distance from the point P with coordinates (1, 2, 1) to the line $r = a + \lambda b$, where $a = i + j + k$ and $b = 2i - j + 3k$.

(6 × 4 = 24 Marks)

PART – IV

Answer any **two** questions. **Each** question carries **15** marks.

32. (a) State Mean Value Theorem. 9

(b) Determine inequalities satisfied by

(i) ln x and

(ii) sin x for suitable ranges of the real variable x. 6

33. (a) Evaluate the integral $I = \int \frac{1}{\sqrt{1-x^2}} dx$. **5**

(b) Show that the value of the integral $I = \int_0^1 \frac{1}{\left[1+x^2+x^3\right]^{3/2}} dx$ lies between 0.810 and 0.882. **10**

34. (a) Find the sum the series $S = 1 + \frac{2}{2} + \frac{3}{2^2} + \frac{4}{2^3} + \ldots$ **5**

(b) Find the power series expansion of $e^{\cos x}$. **10**

35. (a) Find the direction of the line of intersection of the two planes $x + 3y - z = 5$ and $2x - 2y + 4z = 3$. **6**

(b) Find the distance from the point P with coordinates $(1, 2, 3)$ to the plane that contains the points A, B and C having coordinates $(0, 1, 0), (2, 3, 1)$ and $(5, 7, 2)$. **9**

(2 × 15 = 30 Marks)

(Pages : 4)

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Reg. No. : ....

Name : ....

First Semester B.Sc. Degree Examination, March 2023

First Degree Programme under CBCSS

Mathematics

Complementary Course for Physics

MM 1131.1 : MATHEMATICS I – CALCULUS AND SEQUENCES AND SERIES

(2021 Admission Onwards)

Time : 3 Hours Max. Marks : 80

I. Answer the first **ten** questions are compulsory. They carry **1** mark each.

1. Find $\lim_{x \to 1} (x^7 - 2x^5 + 1)^{35}$.

2. What is the value of $\lim_{x \to -\infty} \tan^{-1} x$?

3. Evaluate $\int (x + x^2) dx$.

4. What is the integral of $\tan x$?

5. Find $\int_0^{\pi/2} \frac{\sin x}{5} dx$.

6. Find the area under the curve $y = \sin x$ over the interval $[0, \pi/4]$.

7. Find $\frac{\partial f}{\partial y}$ for the function $f(x, y) = 2x^3y^2 + 2y + 4x$.

P.T.O.

8. Define critical point.

9. Find the general term of the sequence $\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \frac{4}{5}, \ldots$

10. Find the Maclaurin polynomial P_2 for e^x.

(10 × 1 = 10 Marks)

II. Answer any **eight** questions. They questions carry **2** marks each.

11. Find $\lim_{x \to -4} \frac{2x+8}{x^2+x-12}$.

12. For what values of x is there a discontinuity in the graph of $y = \frac{2x+3}{(x-5)(x-6)}$?

13. Find $\frac{dy}{dx}$ if $y = \sec^{-1}(e^x)$.

14. Evaluate $\int \frac{t^2 - 2t^4}{4} dt$.

15. Evaluate $\int \frac{dx}{1+3x^2}$.

16. Evaluate $\int \cos^2 x \, dx$.

17. Describe the level surfaces of $f(x, y, z) = z^2 - x^2 - y^2$.

18. If $f(x, y) = x^2y^3 + x^4y$, find $\frac{\partial^2 f}{\partial y^2}$.

19. Consider the sphere $x^2 + y^2 + z^2 = 1$. Find $\frac{\partial z}{dx}$ and $\frac{\partial z}{\partial y}$ at the point $\left(\frac{2}{3}, \frac{1}{3}, \frac{2}{3}\right)$.

20. Determine whether the sequence $\left\{(-1)^{n+1} \frac{n}{2n+1}\right\}_{n=1}^{+\infty}$ converges or diverges.

21. Determine whether the series $\sum_{k=1}^{\infty} 3^{2k} 5^{1-k}$ converges or diverges.

22. Show that the series $\sum_{k=1}^{\infty} \frac{k}{k+1}$ diverges.

(8 × 2 = 16 Marks)

III. Answer any **six** questions. These questions carry **4** marks each.

23. Find $\lim_{x \to 1} \frac{x-1}{\sqrt{x}-1}$.

24. Find $\lim_{x \to -\infty} \frac{4x^2 - x}{2x^3 - 5}$.

25. Evaluate $\int xe^x \, dx$.

26. Evaluate $\int_1^{\sqrt{2}} \frac{dx}{x^2\sqrt{4-x^2}}$.

27. Let $f(x,y) = y^2e^x + y$. Find f_{xyy}.

28. Given that $z = e^{xy}, x = 2u + v, y = u/v$. Find $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$ using the chain rule.

29. Locate all relative extrema and saddle points of $f(x,y) = 4xy - x^4 - y^4$.

30. Show that the integral test applies and use the integral to determine whether the series $\sum_{k=1}^{\infty} \frac{1}{k^2}$ converge or diverge.

31. Use the comparison test to determine whether the series $\sum_{k=1}^{\infty} \frac{1}{2k^2 + k}$ converge or diverge.

(6 × 4 = 24 Marks)

IV. Answer any **two** questions. These questions carry **15** marks each.

32. (a) Find $\frac{dy}{dx}$ if $y = 3x^8 - 2x^5 + 6x + 1$.

(b) At what points, does the graph of $y = x^3 - 3x + 4$ have a horizontal tangent line?

(c) Find the area of the triangle formed from the coordinate axes and the tangent line to the curve $y = 5x^{-1} - \frac{1}{5}x$ at the point $(5, 0)$.

33. (a) Evaluate $\int \sin^4 x \cos^4 x \, dx$.

(b) Evaluate $\int \tan^2 x \sec^4 x \, dx$.

34. Use Lagrange multipliers to determine the dimensions of a rectangular box, open at the top, having a volume of 32 ft^3 and requiring the least amount of material for its construction.

35. (a) Find the n^{th} Maclaurin polynomial for $\frac{1}{1-x}$ and express it in sigma notation.

(b) Find the n^{th} Taylor polynomial for $\frac{1}{x}$ about $x = 1$ and express it in sigma notation.

(2 × 15 = 30 Marks)

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Reg. No. : ....

Name : ....

First Semester B.Sc. Degree Examination, March 2023

First Degree Programme Under CBCSS

Mathematics

Complementary Course I for Statistics

MM 1131.4 : MATHEMATICS I – DIFFERENTIAL CALCULUS

(2021 Admission onwards)

Time : 3 Hours Max. Marks : 80

I. Answer **all** questions. Each questions carries **1** mark.

1. Determine whether the statement is true or false. If $f(x)$ is cubic polynomial, then $f'(x)$ is a quadratic polynomial.

2. Find x such that $\ln(x+1)=5$.

3. Find $\lim\limits_{x\to 0}\dfrac{2x^2+x}{x}$.

4. State Mean-value theorem.

5. Find $\lim\limits_{x\to +\infty}\dfrac{x}{e^x}$.

6. What is meant by a function is concave up on an interval?

7. Find $\dfrac{\partial z}{\partial y}$ if $z=9xy^2-3x^5y$.

8. Define the partial derivative of $f(x,y)$ with respect to x at (x_0,y_0).

P.T.O.

9. Find all critical points of $f(x) = 3x^3 - 12x$.

10. Determine whether the statement is true or false. If a function f is continuous on $[a,b]$, then f has an absolute maximum on $[a,b]$.

(10 × 1 = 10 Marks)

II. Answer **any eight** questions. **Each** questions carries **2** marks.

11. Find $\frac{d^2y}{dx^2}$ if $y = \ln(x^2+1)$.

12. Find $\frac{dy}{dx}$ if $y = \tan(\sqrt{x})$.

13. Find $\lim_{x \to -3} \frac{3x+9}{x^2+4x+3}$.

14. Find the intervals on which $f(x) = 5 - 4x - x^2$ is concave up.

15. Find the relative extrema of the function $f(x) = x^3 - 4x^2 + 4x$.

16. Prove that $f(x) = \frac{1}{x}$ is decreasing on $(0, +\infty)$.

17. Find $\lim_{x \to 1} \frac{x^2-1}{x^3-1}$.

18. Find the slope of the surface $z = x^2y + 5y^3$ in the x-direction at the point $(1,-2)$.

19. Find $\frac{dz}{dt}$ if $z = 3x^2y^3$; $x = t^4$; $y = t^2$.

20. Show that $\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x}$ if $z = x^2y^3 + x^4y$.

21. Describe the natural domain of $f(x,y,z) = \sqrt{25 - x^2 - y^2 - z^2}$.

22. Describe the level surface of the function $f(x,y,z) = \sqrt{x^2 + y^2 + z^2 - 1}$.

(8 × 2 = 16 Marks)

III. Answer **any six** questions. **Each** questions carries **4** marks.

23. Find the values of x at which $f(x)=\begin{cases} 2x+3 & x \le 4 \\ 7+\dfrac{16}{x} & x > 4 \end{cases}$ is not continuous.

24. Find $\frac{d}{dx}\left(1+x^5 \cot x\right)^{-8}$.

25. Suppose that $s = 1+5t-2t^2$ is the position function of a particle, where s is in meters and t is in seconds. Find the average velocities of the particle over the time intervals

(a) $[0, 2]$ and

(b) $[2, 3]$.

26. Find $\lim_{x\to 0}(1+\sin x)^{\frac{1}{x}}$.

27. Verify Rolle's theorem for $f(x) = x^2 - 8x + 15$ in $[3, 5]$.

28. Find $\frac{dw}{d\theta}$ is $\omega = \sqrt{x^2+y^2+z^2}, x = \cos\theta, y = \sin\theta, z = \tan\theta$.

29. Find the relative extrema of $f(x, y) = x^2 + xy + y^2 - 3x$.

30. Show that the function $z = e^{-t} \sin\left(\frac{x}{c}\right)$ satisfies he Heat equation $\frac{\partial z}{\partial t} = c^2 \frac{\partial^2 z}{\partial x^2}$.

31. If $f(x, y) = x^2 + xy + y^2$, prove that $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y} = 2f(x, y)$.

(6 × 4 = 24 Marks)

IV. Answer any **two** questions. Each questions carries **15** marks.

32. (a) A man has 100ft of fencing to enclose a rectangular garden. Find the dimension of the garden of largest area he can have if he uses all of the fencing.

(b) Find the slope and an equation of the tangent line to the graph $f(x)=x^2$ at the point $(1,1)$.

33. (a) Let f be a differentiable function of one variable and let $\omega=f(u)$, where $u=(x^2+y^2+z^2)^{\frac{1}{2}}$. Show that

$$\left(\frac{\partial w}{\partial x}\right)^2+\left(\frac{\partial w}{\partial y}\right)^2+\left(\frac{\partial w}{\partial z}\right)^2=\left(\frac{dw}{du}\right)^2$$

(b) Find the absolute extrema of $f(x)=\frac{1}{x^2-x}$ on $(0,1)$

34. (a) Find $\frac{\partial z}{\partial u}$ and $\frac{\partial z}{\partial v}$ if $z=e^{xy}, x=2u+v, y=\frac{u}{v}$.

(b) Solve $\frac{e^x-e^{-x}}{2}=1$ for x.

35. (a) Use implicit differentiation to find $\frac{d^2y}{dx^2}$ if $5y^2+\sin y=x^2$.

(b) Use Lagrange multipliers to determine the point on the line $2x-4y=3$ that is closest to the origin.

(2 × 15 = 30 marks)

(Pages : 4)

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Reg. No. : ....

Name : ....

First Semester B.Sc. Degree Examination, March 2023

First Degree Programme under CBCSS

Mathematics

Complementary Course I for Chemistry and Polymer Chemistry

MM 1131.2 : MATHEMATICS I - DIFFERENTIAL CALCULUS AND SEQUENCES AND SERIES

(2021 Admission onwards)

Time : 3 Hours Max. Marks : 80

SECTION – A

All the first **10** questions are compulsory. They carry **1** mark each.

1. Find $\lim_{x\to 2}(x^2 - x + 1)$.

2. State product rule for differentiation.

3. Evaluate $\log_2 5$ in terms of natural logarithms.

4. Find the domain of the function $f(x,y) = \frac{ln(x+y+1)}{y-x}$.

5. Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ if $f(x,y) = x^2y + 5y^3$.

6. Define an inflection point of a function.

P.T.O.

7. Using L'Hôpital's rule, find $\lim_{x \to 0} \frac{\sin 2x}{x}$.

8. State the Extreme-Value Theorem.

9. Evaluate $\sum_{n=0}^{\infty} \frac{(-1)^n 5}{4^n}$.

10. Show that the sequence $\left\{(-1)^{n+1} \frac{1}{n}\right\}$ converges by finding the limit.

(10 × 1 = 10 Marks)

SECTION – B

Answer any **eight** questions. These question carries **2** marks each.

11. Show that the function f defined by $f(x) = \sqrt{4 - x^2}$ is continuous on the closed interval $(-2,2)$.

12. Find the derivative of $f(x) = \frac{2x^2 + x}{x^3 - 1}$

13. Find the derivative of $f(x) = ln\sqrt{x^2 + 1}$.

14. State Rolle's Theorem and verify it for the function $f(x) = x^3 - x$ for $x \in [-1,1]$.

15. Find all critical points of $f(x) = x^3 - 3x + 1$.

16. Evaluate $\lim_{x \to 0^+} x \ln x$.

17. Define level surface for a function $f(x,y,z)$. Describe the level surfaces of $f(x,y,z) = x^2 + y^2 + z^2$.

18. Find the local linear approximation to $f(x,y) = \sqrt{x^2 + y^2}$ at $(3,4)$.

19. Express $\frac{\partial w}{\partial r}$ and $\frac{\partial w}{\partial s}$ in terms of r and s if $w = x + 2y + z^2$, $x = \frac{r}{s}$, $y = r^2 + \ln s, z = 2r$.

20. Find the Maclaurin series for e^x.

21. Find the sum of the series $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$.

22. Use the alternating series test to check the convergence of $\sum_{k=1}^{\infty} (-1)^{k+1} \frac{1}{k}$..

(8 × 2 = 16 Marks)

SECTION – C

Answer any **six** questions. These questions carries **4** marks.

23. Evaluate $\lim_{x \to 1} \frac{x^2 + x - 2}{x^2 - x}$.

24. Evaluate $\frac{d}{dx} \sec^{-1}(5x^4)$.

25. Find the intervals on which the function $f(x) = x^{\frac{4}{3}} - 4x^{\frac{1}{3}}$ is increasing and decreasing.

26. Find $\lim_{x \to 0} (1 + \sin x)^{\frac{1}{x}}$. Let $y = (1 + \sin x)^{\frac{1}{x}}$.

27. Use chain rule to find $\frac{\partial w}{\partial u}$ and $\frac{\partial w}{\partial v}$, where $w = e^{xyz}$, $x = 3u + v$, $y = 3u - v$, $z = u^2 v$.

28. Find the slope of the sphere $x^2 + y^2 + z^2 = 1$ in the y-direction at the points $\left(\frac{2}{3}, \frac{1}{3}, \frac{2}{3}\right)$ and $\left(\frac{2}{3}, \frac{1}{3}, -\frac{2}{3}\right)$.

29. Find the first four Taylor polynomials for ln x about $x = 2$.

30. Test the convergence of the following series

(a) $\sum_{k=1}^{\infty} \frac{k^k}{k!}$

(b) $\sum_{k=1}^{\infty} \frac{1}{2^k - 1}$

31. Show that $|x|$ is continuous everywhere.

(6 × 4 = 24 Marks)

SECTION – D

Answer any **two** questions. These question carries **15** marks.

32. (a) Sketch a graph of $y = \frac{x^2 - 1}{x^3}$ and identify the locations of all asymptotes, intercepts, relative extrema, and inflection points.

(b) Find the slope of circle $x^2 + y^2 = 25$ at the point (3, –4).

33. (a) Use implicit differentiation to find $\frac{dy}{dx}$ if $y^2 = x^2 + \sin xy$.

(b) Find the tangent to the curve $x^3 + y^3 - 9xy = 0$ at the point (2,4).

34. (a) Find the local extreme values of the function $f(x, y) = xy - x^2 - y^2 - 2x - 2y + 4$.

(b) At what point or points on the circle $x^2 + y^2 = 1$ does $f(x, y) = xy$ have an absolute maximum, and what is that maximum?

35. (a) Find the interval of convergence and radius of convergence of $\sum_{k=1}^{\infty} \frac{(x-5)^k}{k^2}$. (6)

(b) Find the values of x for which the power series $\sum_{k=1}^{\infty} k!\, x^k$ converge. (3)

(c) Find the values of x for which the power series $\sum_{k=1}^{\infty} \frac{(-1)^{k-1} x^{2k-1}}{2k-1}$ converge.

(2 × 15 = 30 Marks)