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Reg. No. : ....

Name : ....

Fourth Semester M.Sc. Degree Examination, November 2022

SDE

Mathematics

MM 241 : COMPLEX ANALYSIS – II

(2017 Admission Onwards)

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Time : 3 Hours

Max. Marks : 75

Instructions: 1) Answer either Part A or Part B of each question

2) All questions carry equal marks.

I. (A) State and prove the Arzela-Ascoli theorem. **15**

OR

(B) (a) Let $H(G)$ be the collection of analytic functions on G which can be treated as a subset of $C(G,\mathbb{C})$. If $\{f_n\}$ is a sequence in $H(G)$ and f belongs to $C(G,\mathbb{C})$ such that $f_n \to f$ then prove that f is analytic and $f_n^{(k)} \to f^{(k)}$ for each integer $k \geq 1$. **7**

(b) State and prove Hurwitz's theorem. **8**

II. (A) Let G be a region and let $\{a_j\}$ be a sequence of distinct points in G with no limit point in G. Let $\{m_j\}$ be a sequence of integers. Then prove that there is an analytic function f defined on G whose only zeros are at the points a_j. Also prove that a_j is a zero of f of multiplicity m_j **15**

OR

P.T.O.

(B) (a) Show that $\cos \pi z = \prod_{n=1}^{\infty}\left[1-\frac{4z^2}{(2n-1)^2}\right]$ 7

(b) State and prove the Bohr-Mollerup theorem. 8

III. (A) (a) Prove that for Re $z > 1$, $\zeta(z)\Gamma(z) = \int_0^{\infty}(e^t - 1)^{-1}t^{z-1}dt$ 7

(b) Let γ be a rectifiable curve and let K be a compact set such that $K \cap \{\gamma\} = \phi$. If f is a continuous function on $\{\gamma\}$ and $\in > 0$ then prove that there is a rational function $R(z)$ having all its poles on $\{\gamma\}$ and such that $\left|\int_{\gamma}\frac{f(w)}{w-z}dw - R(z)\right| < \in$ for all z in K. 8

OR

(B) State and prove Mittag- Leffler's theorem. 15

IV. (A) State and prove Schwarz Reflection principle. 15

OR

(B) (a) Let $\gamma : [0,1] \to \mathbb{C}$ be a path from a to b and let $\{(f_t, D_t): 0 \le t \le 1\}$ and $\{(g_t, B_t): 0 \le t \le 1\}$ be analytic continuations along γ such that $[f_0]_a = [g_0]_a$. Then prove that $[f_1]_b = [g_1]_b$. 7

(b) State and prove Monodromy theorem. 8

V. (A) (a) Let G be a region and suppose that u and v are two continuous real valued functions on G with mean value property. Prove that if there is a point a in the extended boundary $\partial_{\infty}G$, $\lim_{z\to a}\sup u(z) \le \lim_{z\to a}\inf v(z)$ then either $u(z) < v(z)$ for all z in G or $u = v$. 7

(b) Let $D = \{z : |z| < 1\}$ and suppose that $f : \partial D \to \mathbb{R}$ is a continuous function. Then prove that there is a continuous function $u : D^{-} \to \mathbb{R}$ such that

(i) $u(z) = f(z)$ for z in ∂D.

(ii) u is hormonic in D. 8

OR

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(B) (a) Derive Jensen's formula. **7**

(b) Let f be an entire function of genus μ. Prove that for each positive number α there is a number r_0 such that for $|z| > r_0$, $|f(z)| < \exp\left(\alpha |z|^{\mu+1}\right)$. **8**

(5 × 15 = 75 Marks)

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