(Pages : 6) **P – 3857**

Reg. No. : ....

Name : ....

Third Semester B.Sc. Degree Examination, January 2023

First Degree Programme under CBCSS

Mathematics

Complementary Course for Chemistry and Polymer Chemistry

MM 1331.2 : MATHEMATICS III — LINEAR ALGEBRA, PROBABILITY THEORY AND NUMERICAL SOLUTIONS

(2021 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the first **ten** questions are compulsory. They carry 1 mark each.

1. Give an example of a square matrix.

2. What is an elementary matrix?

3. Define a regular linear transformation.

4. Define eigen value of a matrix.

5. Find the number of permutations of all the letters of the word ‘Committee’.

6. What is a random variable?

7. Write two properties of normal distribution.

P.T.O.

8. The iterative formula for finding the reciprocal of N is $x_{n+1} =$ ____________

9. Evaluate $\Delta \tan^{-1} x$.

10. State trapezoidal rule.

(10 × 1 = 10 Marks)

SECTION – II

Answer **any eight** questions. These questions carry **2** marks each.

11. Find the rank of the matrix $\begin{bmatrix} 2 & 4 & 6 \\ 4 & 8 & 12 \end{bmatrix}$.

12. Find the value of k for which the system of equations $(3k-8)x+3y+3z=0$, $3x+(3k-8)y+3z=0$, $3x+3y+(3k-8)z=0$ has a nontrivial solution.

13. State Cayley-Hamilton theorem and find the characteristic equation of $\begin{bmatrix} 2 & 1 \\ 3 & 5 \end{bmatrix}$.

14. Find the eigen valve of the matrix $\begin{bmatrix} 2 & 0 \\ 0 & -2 \end{bmatrix}$.

15. Show that for any square matrix A, A and A' have the same eigen values.

16. What is the chance that a leap year selected at random will contain 53 Sundays?

17. Find the probability of getting a king of red colour from a well shuffled deck of 52 cards?

18. Evaluate $p(A/B)$ and $p(B/A)$ given $p(A)=1/4$ and $p(B)=1/3$.

19. In 256 sets of 12 tosses of a coin, in how many cases, one can expect 8 heads and 4 tails?

20. Use a binomial distribution to calculate $P(X=0)$ and $P(X=1)$.

21. Suppose 5 cards are drawn at random from a pack of 52 cards. If all cards are red, find the probability that all of them are hearts.

22. Find a real root of the equation $x^3-2x-5=0$ by the method of false position correct to three decimal places.

23. Evaluate $\sqrt{5}$ by Newton's iteration method.

24. Find the missing term in the table

x	2	3	4	5	6
y	45	49.2	54.1	–	67.4

25. Evaluate $\int_0^6 \frac{dx}{1+x^2}$ by using trapezoidal rule.

26. Find a solution using Simpson's 1/3 rule

x	0	0.1	0.2	0.3	0.4
$f(x)$	1	0.9975	0.9900	0.9776	0.8604

(8 × 2 = 16 Marks)

SECTION – III

Answer **any six** questions. These question carry 4 marks each.

27. Find the inverse of the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$.

28. Find x, y, z and *w* given that $3\begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 5 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 6 & x+y \\ z+w & 5 \end{bmatrix}$.

29. Show that the matrix $\begin{bmatrix} 1/3 & -2/3 & 2/3 \\ 2/3 & -1/3 & -2/3 \\ 2/3 & -2/3 & 1/3 \end{bmatrix}$ is orthogonal.

30. Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$.

31. Verify Cayley – Hamilton theorem for the matrix $A=\begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ and find its inverse.

32. Two cards are drawn in succession from a pack of 52 cards. Find the chance that the first is a king and the second queen if the first card is

(a) replaced (b) not replaced.

33. Three identical boxes contain red and white balls. The first box contains 3 red and 2 white balls, the second box has 4 red and 5 white balls, and the third box has 2 red and 4 white balls. A box is chosen very randomly and a ball is drawn from it. If the ball that is drawn out is red, what will be the probability that the second box is chosen?

34. A die is tossed thrice. A success is "getting 1 or 6" on a toss. Find the mean and variance of the number of successes.

35. Find the cubic polynomial which takes the following values :

x	0	1	2	3
$f(x)$	1	2	1	10

Hence evaluate $f(4)$.

36. If $y_{10}=3, y_{11}=6, y_{12}=11, y_{13}=18, y_{14}=27$, find y_4.

37. Use Trapezoidal rule to estimate the integral $\int_0^2 e^{x^2}\,dx$ taking 10 intervals.

38. Find $y(0.2)$ for $y'=x^2y-1$, $y(0)=1$ with step length 0.1 using Taylor series method.

(6 × 4 = 24 Marks)

SECTION – IV

Answer **any two** questions. These question carry 15 marks each.

39. Reduce the matrix $A=\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$ to normal form and hence find the rank.

40. Investigate the value of λ and μ so that the equations $2x+3y+5z=9$, $7x+3y-2z=8$, $2x+3y+\lambda z=\mu$ have

    (a) No solution (b) a unique solution (c) an infinite number of solutions.

41. A biased coin is tossed till a head appears for the first time

    (a) What is the probability that the number of required tosses is odd.

    (b) Two persons A and B toss an unbiased coin alternatively on the understanding that the first who gets the head wins. if A starts the game, find their respective chance of winning.

42. A random variable X has the following probability function :

x	0	1	2	3	4	5	6	7
$p(x)$	0	k	2k	2k	3k	k^2	$2k^2$	$7k^2+k$

    (a) Find the value of k.

    (b) Evaluate

        (i) $P(X<6)$

        (ii) $P(X\geq 6)$ and

        (iii) $P(0<X<5)$.

43. Using Newton's iterative method, find the real root of the equation $3x = \cos x + 1$.

44. Apply Gauss-Jordan method to solve the equations

$x + y + z = 9, 2x - 3y + 4z = 13, 3x + 4y + 5z = 40$.

(2 × 15 = 30 Marks)

(Pages : 6) **P – 3812**

Reg. No. : ....

Name : ....

Third Semester B.A. Degree Examination, January 2023

First Degree Programme under CBCSS

Mathematics

Complementary Course for Economics

MM 1331.5 : MATHEMATICS FOR ECONOMICS – III

(2021 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the first **ten** questions are compulsory. Each question carries **1** marks.

1. Find $\int x\cdot dx$.

2. Find $\int 3e^{x} dx$.

3. Evaluate $\int 5x^{4} dx$.

4. Find $\int \sqrt{t}\, dt$.

5. Find $\int (x^{2}-1)\, dx$.

6. Define identity matrix.

7. What is the transpose of $\begin{bmatrix} 2 & 1 \\ 4 & 3 \\ 1 & 0 \end{bmatrix}$.

8. Give an example of a diagonal matrix.

9. Find $\begin{bmatrix} 1 & -2 & 4 \\ 2 & 5 & -3 \end{bmatrix} + \begin{bmatrix} -1 & -5 & -6 \\ 0 & 1 & 3 \end{bmatrix}$.

10. Find $\begin{vmatrix} 3 & -2 \\ 1 & 0 \end{vmatrix}$.

(10 × 1 = 10 Marks)

SECTION – II

Answer any **eight** questions. Each question carries **2** marks.

11. Find $\int x\, e^x\, dx$.

12. Find $\int (x+1)^2 dx$.

13. Find $\int (x^5 - 3x) dx$.

14. Integrate $\frac{4x^2 + 2 + \sqrt{x}}{x^2}$.

15. Evaluate $\int \left(5e^x + \frac{3}{x^2} \right) dx$.

16. Find $\int \sqrt{x^3}\, dx$.

17. Find $\int (3x)^{-4} dx$.

18. Find $\int_0^1 x^3\, dx$.

19. If $A=\begin{bmatrix}-1 & 2\\ 2 & -1\end{bmatrix}$ and $B=\begin{bmatrix}3\\ 1\end{bmatrix}$, find AB.

20. Define symmetric matrix. Give an example.

21. If $A=\begin{bmatrix}2 & 3\\ 4 & 5\end{bmatrix}$, $B=\begin{bmatrix}3 & 1\\ 2 & 5\end{bmatrix}$, find $(A+B)'$.

22. Let $A=\begin{bmatrix}3 & 1\\ 0 & 2\end{bmatrix}$. Verify whether $B=\frac{1}{6}\begin{bmatrix}2 & -1\\ 0 & 3\end{bmatrix}$ is the inverse of A.

23. Check whether the matrix $\begin{bmatrix}6 & 3\\ 8 & 4\end{bmatrix}$ is singular.

24. Define a triangular matrix. Give an example.

25. If $A=\begin{bmatrix}2 & -1\\ 4 & 2\end{bmatrix}$, find A^2.

26. Solve by Cramer's rule.

$$3x+4y=5$$
$$3x-4y=2$$

(8 × 2 = 16 Marks)

SECTION – III

Answer any **six** questions. Each question carries **4** marks.

27. Find $\int \frac{3-x^2}{x^2}dx$.

28. Integrate $(5x+7)^8$.

29. Integrate x^2e^x.

30. Integrate $2x(x^2+1)$.

31. Evaluate $\int_1^4 \sqrt{x}\, dx$.

32. Give the marginal cost function $f'(x)=2+x+x^2$, find the total cost function when fixed cost is 50 units; x being the output produced.

33. Compute $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 & 0 \\ 1 & 0 & 0 \\ -1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 5 \\ 6 \end{bmatrix}$.

34. If $A=\begin{bmatrix} 9 & 1 \\ 4 & 3 \end{bmatrix}$ and $B=\begin{bmatrix} 1 & 5 \\ 7 & 12 \end{bmatrix}$, find the matrix X such that $3A+5B+2X=0$.

35. Let $A=\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B=\begin{bmatrix} 0 & -1 \\ 6 & 7 \end{bmatrix}$, find $B'A'$.

36. Verify whether the matrix $=\begin{bmatrix} -6 & 3 & 5 \\ -10 & 2 & 8 \\ 5 & 2 & 3 \end{bmatrix}$ possess an inverse.

37. If $A=\begin{bmatrix} 3 & 1 \\ 2 & 5 \end{bmatrix}$, find $A^2-5A+7I$.

38. Find the solution of the following system of equations by Cramer's rule

$$7x-5y=11$$
$$3x+2y=13$$

(6 × 4 = 24 Marks)

SECTION – IV

Answer any **two** questions. **Each** question carries **15** marks.

39. (a) Evaluate $\int\left(\frac{1}{x^2}+\frac{4}{x\sqrt{x}}+2\right)dx$. **5**

(b) Evaluate $\int 3x^2e^{5x}\,dx$. **5**

(c) Evaluate $\int_{-1}^{3}(1+3x-x^2)dx$. **5**

40. If $MR=16-x^2$, find the total revenue, average revenue and demand. **15**

41. If the marginal revenue function is : $p_m=\left\{\frac{ab}{(x+b)^2}-C\right\}$, show that $p=\left\{\frac{ab}{(x+b)}-C\right\}$ is the demand law. **15**

42. (a) If $A=\begin{bmatrix}7 & 1\\ 0 & 4\end{bmatrix}$, $B=\begin{bmatrix}2 & 1\\ 1 & 3\end{bmatrix}$, $C=\begin{bmatrix}1 & 2\\ 4 & 1\end{bmatrix}$ and $D=\begin{bmatrix}5 & 2\\ -1 & 3\end{bmatrix}$. Find $(A+B)(C+D)$. **7**

(b) Find the inverse of the matrix $\begin{bmatrix}1 & 2 & 3\\ 1 & 3 & 5\\ 1 & 5 & 12\end{bmatrix}$. **8**

43. (a) Find the value of $\begin{vmatrix}1 & 3 & 1\\ 2 & 5 & 4\\ 6 & 1 & 0\end{vmatrix}$. **7**

(b) Compute $\begin{bmatrix}1 & 2 & -3\\ -4 & 3 & 0\end{bmatrix}\begin{bmatrix}-1 & 0 & 2 & 3 & 1\\ 2 & -1 & 4 & 0 & -3\\ -3 & 0 & 1 & 2 & -1\end{bmatrix}\begin{bmatrix}1\\ 0\\ 0\\ 0\\ 0\end{bmatrix}$. **8**

44. (a) Find the adjoint of $A=\begin{bmatrix}-4 & -3 & -3\\ 1 & 0 & 1\\ 4 & 4 & 3\end{bmatrix}$. **7**

(b) Solve the following set of equations using Cramer's rule. **8**

$$7x-y-z=0$$
$$10x-2y+z=8$$
$$6x+3y-2z=7$$

(2 × 15 = 30 Marks)

(Pages : 6) **P – 3812**

Reg. No. : ....

Name : ....

Third Semester B.A. Degree Examination, January 2023

First Degree Programme under CBCSS

Mathematics

Complementary Course for Economics

MM 1331.5 : MATHEMATICS FOR ECONOMICS – III

(2021 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the first **ten** questions are compulsory. Each question carries **1** marks.

1. Find $\int x \, dx$.

2. Find $\int 3e^x dx$.

3. Evaluate $\int 5x^4 \, dx$.

4. Find $\int \sqrt{t} \, dt$.

5. Find $\int (x^2 - 1) \, dx$.

6. Define identity matrix.

P.T.O.

7. What is the transpose of $\begin{bmatrix} 2 & 1 \\ 4 & 3 \\ 1 & 0 \end{bmatrix}$.

8. Give an example of a diagonal matrix.

9. Find $\begin{bmatrix} 1 & -2 & 4 \\ 2 & 5 & -3 \end{bmatrix} + \begin{bmatrix} -1 & -5 & -6 \\ 0 & 1 & 3 \end{bmatrix}$.

10. Find $\begin{vmatrix} 3 & -2 \\ 1 & 0 \end{vmatrix}$.

(10 × 1 = 10 Marks)

SECTION – II

Answer any **eight** questions. Each question carries **2** marks.

11. Find $\int x\, e^x\, dx$.

12. Find $\int (x+1)^2 dx$.

13. Find $\int (x^5 - 3x) dx$.

14. Integrate $\frac{4x^2 + 2 + \sqrt{x}}{x^2}$.

15. Evaluate $\int \left(5e^x + \frac{3}{x^2}\right) dx$.

16. Find $\int \sqrt{x^3}\, dx$.

17. Find $\int (3x)^{-4} dx$.

18. Find $\int_0^1 x^3 \, dx$.

19. If $A=\begin{bmatrix} -1 & 2 \\ 2 & -1 \end{bmatrix}$ and $B=\begin{bmatrix} 3 \\ 1 \end{bmatrix}$, find AB.

20. Define symmetric matrix. Give an example.

21. If $A=\begin{bmatrix} 2 & 3 \\ 4 & 5 \end{bmatrix}$, $B=\begin{bmatrix} 3 & 1 \\ 2 & 5 \end{bmatrix}$, find $(A+B)'$.

22. Let $A=\begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}$. Verify whether $B=\frac{1}{6}\begin{bmatrix} 2 & -1 \\ 0 & 3 \end{bmatrix}$ is the inverse of A.

23. Check whether the matrix $\begin{bmatrix} 6 & 3 \\ 8 & 4 \end{bmatrix}$ is singular.

24. Define a triangular matrix. Give an example.

25. If $A=\begin{bmatrix} 2 & -1 \\ 4 & 2 \end{bmatrix}$, find A^2.

26. Solve by Cramer's rule.

$$3x+4y=5$$
$$3x-4y=2$$

(8 × 2 = 16 Marks)

SECTION – III

Answer any **six** questions. Each question carries **4** marks.

27. Find $\int \frac{3-x^2}{x^2} dx$.

28. Integrate $(5x+7)^8$.

29. Integrate x^2e^x.

30. Integrate $2x(x^2+1)$.

31. Evaluate $\int_1^4 \sqrt{x}\, dx$.

32. Give the marginal cost function $f'(x)=2+x+x^2$, find the total cost function when fixed cost is 50 units; x being the output produced.

33. Compute $\begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{bmatrix} \begin{bmatrix} 2 & 3 & 0 \\ 1 & 0 & 0 \\ -1 & 1 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 5 \\ 6 \end{bmatrix}$.

34. If $A=\begin{bmatrix} 9 & 1 \\ 4 & 3 \end{bmatrix}$ and $B=\begin{bmatrix} 1 & 5 \\ 7 & 12 \end{bmatrix}$, find the matrix X such that $3A+5B+2X=0$.

35. Let $A=\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$ and $B=\begin{bmatrix} 0 & -1 \\ 6 & 7 \end{bmatrix}$, find $B'A'$.

36. Verify whether the matrix $=\begin{bmatrix} -6 & 3 & 5 \\ -10 & 2 & 8 \\ 5 & 2 & 3 \end{bmatrix}$ possess an inverse.

37. If $A=\begin{bmatrix} 3 & 1 \\ 2 & 5 \end{bmatrix}$, find $A^2-5A+7I$.

38. Find the solution of the following system of equations by Cramer's rule

$$7x-5y=11$$
$$3x+2y=13$$

(6 × 4 = 24 Marks)

SECTION – IV

Answer any **two** questions. **Each** question carries **15** marks.

39. (a) Evaluate $\int\left(\frac{1}{x^2}+\frac{4}{x\sqrt{x}}+2\right)dx$. **5**

(b) Evaluate $\int 3x^2e^{5x}\,dx$. **5**

(c) Evaluate $\int_{-1}^{3}(1+3x-x^2)dx$. **5**

40. If $MR=16-x^2$, find the total revenue, average revenue and demand. **15**

41. If the marginal revenue function is : $p_m=\left\{\frac{ab}{(x+b)^2}-C\right\}$, show that $p=\left\{\frac{ab}{(x+b)}-C\right\}$ is the demand law. **15**

42. (a) If $A=\begin{bmatrix}7 & 1\\0 & 4\end{bmatrix}$, $B=\begin{bmatrix}2 & 1\\1 & 3\end{bmatrix}$, $C=\begin{bmatrix}1 & 2\\4 & 1\end{bmatrix}$ and $D=\begin{bmatrix}5 & 2\\-1 & 3\end{bmatrix}$. Find $(A+B)(C+D)$. **7**

(b) Find the inverse of the matrix $\begin{bmatrix}1 & 2 & 3\\1 & 3 & 5\\1 & 5 & 12\end{bmatrix}$. **8**

43. (a) Find the value of $\begin{vmatrix}1 & 3 & 1\\2 & 5 & 4\\6 & 1 & 0\end{vmatrix}$. **7**

(b) Compute $\begin{bmatrix}1 & 2 & -3\\-4 & 3 & 0\end{bmatrix}\begin{bmatrix}-1 & 0 & 2 & 3 & 1\\2 & -1 & 4 & 0 & -3\\-3 & 0 & 1 & 2 & -1\end{bmatrix}\begin{bmatrix}1\\0\\0\\0\\0\end{bmatrix}$. **8**

44. (a) Find the adjoint of $A = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$. 7

(b) Solve the following set of equations using Cramer's rule. 8

$$7x - y - z = 0$$
$$10x - 2y + z = 8$$
$$6x + 3y - 2z = 7$$

(2 × 15 = 30 Marks)

(Pages : 6) **P – 3856**

Reg. No. : ....

Name : ....

Third Semester B.Sc. Degree Examination, January 2023

First Degree Programme under CBCSS

Mathematics

Complementary Course for Chemistry and Polymer Chemistry

MM 1331.2 – Mathematics III : LINEAR ALGEBRA, PROBABILITY THEORY AND NUMERICAL METHODS

(2019 – 2020 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – I

All the first **ten** questions are compulsory. Each question carries **1** mark.

1. Define the rank of a matrix.

2. Evaluate the determinant $\begin{vmatrix} 0 & a & -b \\ -a & 0 & c \\ b & -c & 0 \end{vmatrix}$.

3. What is the magnitude of a vector?

4. Define Kronecker δ.

P.T.O.

5. What is the scalar product of two vectors?

6. What is the sample space of an event?

7. There are 10 chairs in a row and 8 people to be seated. In how many ways can this be done?

8. Write the expression for variance of a random variable x and explain the terms.

9. Write Baye's formula for conditional probability.

10. What is an algebraic equation?

(10 × 1 = 10 Marks)

SECTION – II

Answer **any eight** questions Each question carries **2** marks.

11. Find the rank of the matrix $\begin{pmatrix} 1 & 1 & 2 \\ 2 & 4 & 6 \\ 3 & 2 & 5 \end{pmatrix}$.

12. Evaluate the determinant $\begin{vmatrix} 1 & -5 & 2 \\ 7 & 3 & 4 \\ 2 & 1 & 5 \end{vmatrix}$.

13. Find the cross product of the vectors $A = 2i + j - k$ and $B = i + 3j - 2k$.

14. Find the symmetric equation of the line through $(1, -1, -5)$ and $(2, -3, -3)$.

15. Find the product of A and B if $A=\begin{pmatrix} 4 & 2 \\ -3 & 1 \end{pmatrix}$ and $B=\begin{pmatrix} 1 & 5 & 3 \\ 2 & 7 & -4 \end{pmatrix}$.

16. Define linear functions.

17. Find the probability that a single card drawn from a shuffled deck of cards will be either a diamond or a king.

18. Two dice are rolled. What is the probability that the sum is ≥ 10?

19. Define mutually exclusive events.

20. If $P(A) = 0.07755$, $P(A\ B) = 0.038$, find $P_A(B)$.

21. What is the probability that a number $n, 1 \leq n \leq 99$. is divisible by both 6 and 10?

22. A club consists of 50 members. In how many ways can a president, vice president, secretary and treasurer be chosen?

23. Write Newton-Raphson iteration formula.

24. Write an iteration scheme for finding the square root of X.

25. What is binary chopping?

26. Evaluate the integral $I = \int_0^1 \frac{1}{1+x^2}\, dx$ using the trapezium rule.

(8 × 2 = 16 Marks)

SECTION – III

Answer **any six** questions. Each question carries **4** marks.

27. Write and row reduce the augmented matrix for the equations :

$$x-y+4z=5$$
$$2x-3y+8z=4$$
$$x-2y+4z=9$$

28. Evaluate the determinant $D=\begin{vmatrix} 4 & 3 & 0 & 1 \\ 9 & 7 & 2 & 3 \\ 4 & 0 & 2 & 1 \\ 3 & -1 & 4 & 0 \end{vmatrix}$.

29. Using Cramer's rule solve the set of equations :

$$2x+3y=3$$
$$x-2y=5$$

30. Find the equation of a line through $(1, 0, -2)$ and perpendicular to the plane $3x-4y+z+6=0$.

31. Find the distance between the lines $r=i-2j+(i-k)t$ and $r=2j-k+(j-i)t$.

32. Which is the most probable sum in a toss of two dice? what is its probability?

33. Two students are working separately on the same problem. If the first student has probability $\frac{1}{2}$ of solving it and the second student has probability $\frac{3}{4}$ of solving it. what is the probability that atleast one of them solves it.?

34. Find the coefficient of x^8 in the binomial expansion of $(1+x)^{15}$.

35. Using Baye's formula find the probability of all heads in three tosses of a coin if you know that atleast one is a head?

36. Evaluate $I=\int_0^2\left(x^2-3x+4\right)dx$ using trapezium rule with $h=0.5$.

37. Evaluate $I=\int_0^1\frac{1}{1+x^2}dx$ using Gaussian integration.

38. Find an explicit formula that will generate a random number y distributed on $(-\infty,\infty)$ according to the Cauchy distribution $f(y)\,dy=\left(\frac{a}{\pi}\right)\frac{dy}{a^2+y^2}$, given a random number ξ uniformly distributed on $(0,1)$.

(6 × 4 = 24 Marks)

SECTION – IV

Answer **any two** questions. Each question carries **15** marks.

39. Diagonalize $H=\begin{pmatrix}2 & 3-i\\ 3+i & -1\end{pmatrix}$.

40. Find the rotation matrix C if the quadratic surface $x^2+6xy-2y^2-2yz+z^2=24$ is rotated to principal axis.

41. A preliminary test is customarily given to the students at the beginning of a certain course.

The following data are accumulated after several years :

(a) 95% of the students pass the course,

(b) 96% of the students who pass the course also passed the preliminary test.

(c) 25% of the students who fail the course passed the preliminary test.

What is the probability that a student who failed the preliminary test will pass the course?

42. Derive the Poison density function $P_n = \frac{\mu^n}{n!} e^{-\mu}$.

43. Solve the simultaneous equations

$$x_1 + 6x_2 - 4x_3 = 8$$
$$3x_1 - 20x_2 + x_3 = 12$$
$$-x_1 + 3x_2 + 5x_3 = 3$$

using Gaussian elimination.

44. Explain any three Monte Carlo methods.

(2 × 15 = 30 Marks)

(Pages : 4)

P – 3811

Reg. No. : ....

Name : ....

Third Semester B.A. Degree Examination, January 2023

First Degree Programme under CBCSS

Mathematics

Complementary Course for Economics

MM 1331.5 : MATHEMATICS FOR ECONOMICS – III

(2019 – 2020 Admission)

Time : 3 Hours | Max. Marks : 80

PART A

Answer **all** the questions. **Each** question carries **1** mark.

1. Evaluate the integral $\int e^{x+3}\,dx$.
2. Find the anti derivative of $\sqrt{x}$.
3. Evaluate the integral $\int (3x^4 + 5x^2 - 2)\,dx$.
4. Evaluate the integral $\int (1+x)^{2/3}\,dx$.
5. State the constant multiple property of integrals.
6. Write Taylor formula.
7. Write the geometric series with $a = 1/9$ and $r = 1/3$.
8. Write the Maclaurin series expansion of e^x.
9. Determine whether the series $1 + 3 + 3^2 + 3^3 + \ldots$ is convergent or divergent.
10. The sum of n terms of a series is $\frac{n}{2n+10}$, Find the sum of the series.

(10 × 1 = 10 Marks)

P.T.O.

PART B

Answer **any eight** questions. Each question carries 2 marks.

11. Calculate the area under the parabola $y = x^2$ over the interval $[0, 1]$.

12. Find the Integral $\int \frac{x^4 + 3x - 2}{x} dx$.

13. Evaluate $\int \frac{x}{x^2 + 1} dx$.

14. Use integration by parts to evaluate $\int xe^x \, dx$.

15. Evaluate $\int_0^{10} \frac{1}{2x + 3} dx$.

16. Find the total cost function if the cost of zero output is c and the marginal cost of output x is $\pi_m = ax + b$.

17. Find the sum of the series $1 - \frac{1}{2} + \frac{1}{4} - \frac{1}{8} + \ldots$

18. Find the binomial series for the function $(1 + x)^4, |x| < 1$.

19. Find the sum of the series $\sum_{n=1}^{\infty} \frac{1}{(n+1)!}$.

20. Show that the accumulated value of a constant income stream a will be $\frac{a}{r}(e^{rx} - 1)$.

21. Find a power series representation for $\ln(1 - x)$ on $(-1, 1)$.

22. Use Simpson's rule with $n = 4$ to approximate $\int_0^1 \frac{dx}{x + 1}$.

23. If $f(x) = f'(x)$, what is $f(x)$?

24. Find the anti-derivative of $\frac{ax + b}{\sqrt{x}} dx$.

25. Describe the concept of capitalization.

26. Find the sum of $1-\frac{1}{2!}+\frac{1}{4!}-\frac{1}{6!}+\ldots$.

(8 × 2 = 16 Marks)

PART C

Answer **any six** questions. Each questions carries **4** marks.

27. Show that

(a) $\int \frac{e^x}{1+e^x}dx = \log(1+e^x)+c$

(b) $\int \frac{e^x+e^{-x}}{e^x-e^{-x}}dx = \log(e^x-e^{-x})+c$

28. The marginal cost of producing x units of some commodity is $1+x+3x^2$ and fixed costs are 150. Find the total cost function.

29. Find the area under the straight line $y = ax + b$ above the x-axis between the coordinates $x = 0$ and $x = 1$.

30. Use the trapezoidal rule with $n = 4$ to estimate $\int_1^2 x^2\,dx$.

31. Expand $\ln(2 + x)$ around $x = 0$.

32. Find the sum of the series $\sum_{n=1}^{\infty}\frac{3^{n-1}-1}{6^{n-1}}$

33. Find the Maclaurin series for $\sin x$.

34. Show that the length of life of the capital good $f(t)=b\sqrt{t}$, b a constant, varies inversely with respect to the rate of interest.

35. Find a power series representation for $f(x)=\frac{1}{1-x^2}$ on (–1, 1) by differentiating a power series representation of $f(x)=\frac{1}{1-x}$.

36. Evaluate $\int \log x \, dx$.

37. If $y = \int_0^{x^3} \cos t^2 \, dx$, find $\frac{dy}{dx}$.

38. Show that the area between the rectangular hyperbola $xy = \alpha^2$ and the x-axis and between the ordinates at $x = a$ and $x = b$ is $\alpha^2 \log\frac{b}{a}$, if a and b have the same sign.

(6 × 4 = 24 Marks)

PART D

Answer **any two** questions. Each questions carries **15** marks.

39. Evaluate the following integrals

(a) $\int x \sin x \, dx$

(b) $\int x^3 e^x \, dx$

(c) $\int 5^{2x+3} \, dx$.

40. Using Simpson's rule find the area under the curve $y = e^{-x^2}$ above the x-axis and between the ordinates $x = 0$ and $x = 2$ by dividing the interval into 10 equal parts.

41. (a) Show that the Taylor series generated by e^x at $x = 0$ converges to $f(x)$ for every real x.

(b) Find the Taylor series generated by $f(x) = 1/x$ at $a = 2$.

42. Explain Domar's model of public debt and national income.

43. For $|x| < 1$, show that $(1+x)^m = 1 + \sum_{k=1}^{\infty} mC_k \, x^k$.

44. State and prove fundamental theorem of Calculus, Part 1.

(2 × 15 = 30 Marks)

(Pages : 4) **P – 3803**

Reg. No. : ....

Name : ....

Third Semester B.A. Degree Examination, January 2023.

First Degree Programme under CBCSS

Mathematics

Complementary Course for Economics

MM 1331.5 : MATHEMATICS FOR ECONOMICS III

(2013–2018 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions. Each question carries **1** mark.

1. Evaluate the integral $\int e^{x+3}dx$.

2. Find the anti derivative of $\sqrt{x}$.

3. Evaluate the integral $\int (3x^4 + 5x^2 - 2)dx$.

4. Evaluate the integral $\int (1+x)^{2/3}dx$.

5. State the constant multiple property of integrals.

6. Write Taylor formula.

7. Write the geometric series with $a = 1/9$ and $r = 1/3$.

P.T.O.

8. Write the Maclaurin series expansion of e^x.

9. Determine whether the series $1+3+3^2+3^3+.....$ is convergent or divergent.

10. The sum of n terms of a series is $\frac{n}{2n+10}$. Find the sum of the series.

(10 × 1 = 10 Marks)

SECTION – B

Answer **any eight** questions. Each question carries **2** marks.

11. Calculate the area under the parabola $y = x^2$ over the interval [0,1].

12. Find the integral $\int \frac{x^4+3x-2}{x} dx$.

13. Evaluate $\int \frac{x}{x^2+1} dx$.

14. Use integration by parts to evaluate $\int xe^x \, dx$.

15. Evaluate $\int_0^{10} \frac{1}{2x+3} dx$

16. Find the total cost function if the cost of zero output is c and the marginal cost of output x is $\pi_m = ax+b$. .

17. Find the sum of the series $1-\frac{1}{2}+\frac{1}{4}-\frac{1}{8}+....$

18. Find the binomial series for the function $(1+x)^4, |x|<1$.

19. Find the sum of the series $\sum_{n=1}^{\infty} \frac{1}{(n+1)!}$.

20. Show that the accumulated value of a constant income stream a will be $\frac{a}{r}(e^{rx}-1)$.

21. Find a power series representation for $ln(1-x)$ on $(-1,1)$.

22. Use Simpson's rule with $n=4$ to approximate $\int_0^1 \frac{dx}{x+1}$.

(8 × 2 = 16 Marks)

SECTION – C

Answer **any six** questions. Each question carries **4** mark.

23. Show that

(a) $\int \frac{e^x}{1+e^x}dx = \log(1+e^x)+c$

(b) $\int \frac{e^x+e^{-x}}{e^x-e^{-x}}dx = \log(e^x-e^{-x})+c$

24. The marginal cost. of producing x units of sonic commodity is $1+x+3x^2$ and fixed costs are 150. Find the total cost function.

25. Find the area under the straight line $y=ax+b$ above the x-axis between the coordinates $x=0$ and $x=1$.

26. Use the trapezoidal rule with $n=4$ to estimate $\int_0^1 x^2dx$.

27. Expand $ln(2+x)$ around $x=0$.

28. Find the sum of the series $\sum_{n=1}^{\infty}\frac{3^{n-1}-1}{6^{n-1}}$.

29. Find the Maclaurin series for $\sin x$.

30. Show that the length of life of the capital good $f(t)=b\sqrt{t}, b$ a constant, varies inversely with respect to the rate of interest.

31. Find a power series representation for $f(x)=\frac{1}{1-x^2}$ on $(-1,1)$ by differentiating a power series representation of $f(x)=\frac{1}{1-x}$.

(6 × 4 = 24 Marks)

SECTION – D

Answer **any two** question. Each question carries **15** marks.

32. Evaluate the following integrals

(a) $\int x \sin x \, dx$

(b) $\int x^3 e^x \, dx$

(c) $\int 5^{2x+3} dx$

33. Using Simpson's rule find the area under the curve $y=e^{-x^2}$ above the x-axis and between the ordinates $x=0$ and $x=2$ by dividing the interval into 10 equal parts.

34. (a) Show that the Taylor series generated by e^x at $x=0$ converges to $f(x)$ for every real x.

(b) Find the Taylor series generated by $f(x)=1/x$ at $a=2$. .

35. Explain Domar's model of public debt and national income.

(2 × 15 = 30 Marks)

(Pages : 4) **P – 3825**

Reg. No. : ....

Name : ....

Third Semester B.Sc. Degree Examination, January 2023

First Degree Programme under CBCSS

Mathematics

Core Course

MM 1341 : ELEMENTARY NUMBER THEORY AND CALCULUS – I

(2018 Admission)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions.

1. For every positive integer n, find n consecutive integers that are composite numbers.

2. Prove that there are infinitely many primes.

3. State Dirichlet's theorem.

4. State the Pigeonhole principle.

5. If $r(t) = t^2 i + e^t j - (2\cos \pi t)k$, find $r'(t)$.

6. Prove that a straight line has zero curvature at every point.

7. Evaluate: $\int_0^2 r(t)dt$, where $r(t) = 2ti + 3t^2 j$.

P.T.O.

8. If f is a function of x, y and z, what is the gradient of f?

9. State the chain rule for partial derivatives if $z = f(x, y), x = x(u), y = y(u)$.

10. Let $f(x, y) = y^2 e^x + y$. Evaluate f_{xyy}.

(10 × 1 = 10 Marks)

SECTION – B

Answer any **eight** questions.

11. Using recursion, evaluate, (18, 30, 60, 75, 132).

12. Derive a necessary and sufficient condition for two positive integers to be relatively prime.

13. Prove that (a, b) = (a, a – b).

14. Find the number of positive integers ≤ 2076 and divisible by neither 4 nor 5.

15. If $r(t)$ is a differentiable vector - valued function in 2 – space or 3-space and $\|r(t)\|$ is constant for all t, then show that $r(t)$ and $r'(t)$ are orthogonal vectors for all t.

16. State the Newton's laws of universal gravitation.

17. Show that the circle of radius a which centred at the origin has constant curvature $\frac{1}{a}$.

18. Evaluate the unit tangent vector to the graph of $r(t) = t^2 i + t^3 j$ at the point where $t = 2$.

19. Estimate an equation for the tangent plane and parametric equations for the normal line to the surface $z = x^2 y$ at the point (2, 1, 4).

20. Find the directional derivative of $f(x, y, z) = x^2 y - yz^3 + z$ at (1, –2, 0) in the direction of the vector $a = 2i + j - 2k$.

21. Evaluate $f_x(1, 3)$ and $f_y(1, 3)$ by finding $f_x(x, y)$ and $f_y(x, y)$ where $f(x,y)=2x^3y^2+2y+4x$.

22. Prove that $f(x, y)= x^2 + y^2$ is differentiable at the origin.

(8 × 2 = 16 Marks)

SECTION – C

Answer any **six** questions.

23. Let *e* denote the highest power of 2 that divides *n!* and *b* the number of 1s in the binary representation of *n*. Then show that $n = e + b$.

24. Show that the gcd of the positive integers *a* and *b* is a linear combination of *a* and *b*.

25. Show that 3, 5 and 7 are the only three consecutive odd integers that are primes.

26. Find parametric equations of the tangent line to the circular helix $x=\cos t$, $y=\sin t$, $z=t$ where $t = t_0$, and use that result to find parametric equations for the tangent line at the point $t=\pi$.

27. Find the curvature of the ellipse with vector equation $r=2\cos t\, i+3\sin t\, j$, $(0\le t\le 2\pi)$ at the end points of the major and minor axes.

28. Derive Kepler's third law.

29. Find the slope of the sphere $x^2+y^2+z^2=1$ in the y – direction at the points $\left(\frac{2}{3},\frac{1}{3},\frac{2}{3}\right)$ and $\left(\frac{2}{3},\frac{1}{3},-\frac{2}{3}\right)$.

30. Verify whether the function $z=e^x\sin y+e^y\cos x$ satisfies the equation $\frac{\partial^2 z}{\partial x^2}+\frac{\partial^2 z}{\partial y^2}=0$.

31. Locate all relative extrema and saddle points of $f(x, y)=3x^2-2xy+y^2-8y$.

(6 × 4 = 24 Marks)

SECTION – D

Answer any **two** questions.

32. (a) Prove that there is no polynomial $f(n)$ with integral coefficients that will produce primes for all integers n.

    (b) Find the general solution of the LDE $6x + 8y + 12z = 10$.

33. (a) State and prove the division algorithm.

    (b) Show that the number of leap years l after 1600 and not exceeding a given year y is given by $l = [y/4] - [y/100] + [y/400] - 388$.

34. Suppose that a particle moves through 3 – space so that its position vector at time t is $r(t) = ti + t^2 j + t^3 k$.

    (a) Find the scalar tangential and normal components of acceleration at time t.

    (b) Find the scalar tangential and normal components of acceleration at time $t = 1$.

    (c) Find the vector tangential and normal components of acceleration at time $t = 1$.

    (d) Find the curvature of the path at the point where the particle is located at time $t = 1$.

35. Find the points on the sphere $x^2 + y^2 + z^2 = 36$ that are closest to and farthest from the point (1, 2, 2).

(2 × 15 = 30 Marks)

(Pages : 6) **P – 3829**

Reg. No. : ....

Name : ....

Third Semester B.Sc. Degree Examination, January 2023

First Degree Programme under CBCSS

Mathematics

Core Course

MM 1341 : ELEMENTARY NUMBER THEORY AND CALCULUS – I

(2019 Admission Onwards)

Time : 3 Hours Max. Marks : 80

SECTION – A

Answer **all** questions.

1. For every positive integer n, find n consecutive integers that are composite numbers.

2. Prove that there are infinitely many primes.

3. State Dirichlet's theorem.

4. State the Pigeonhole principle.

5. If $r(t) = t^2 i + e^t j - (2 - \cos \pi t)k$, find $r'(t)$.

6. Prove that a straight line has zero curvature at every point.

P.T.O.

7. Evaluate: $\int_0^2 r(t)\,dt$, where $r(t) = 2ti + 3t^2 j$.

8. If f is a function of x, y and z, what is the gradient of f?

9. State the chain rules for partial derivatives.

10. Let $f(x, y) = y^2 e^x + y$. Evaluate f_{xyy}.

(10 × 1 = 10 Marks)

SECTION – B

Answer any **eight** questions.

11. Show that every composite number n has a prime factor $\le \lfloor \sqrt{n} \rfloor$.

12. Using recursion, evaluate, (18, 30, 60, 73,132).

13. Derive a necessary and sufficient condition for two positive integers to be relatively prime.

14. Prove that $(a, b) = (a, a - b)$.

15. Find the number of positive integers ≤ 2076 and divisible by neither four nor five.

16. Write a short note on twin primes.

17. If $r(t)$ is a differentiable vector – valued function in 2-space or 3-space and $\|r(t)\|$ is constant for all t, then show that $r(t)$ and $r'(t)$ are orthogonal vectors for all t.

18. State any two rules of integration of vector valued functions.

19. State the Newton's laws of universal gravitation.

20. Show that the circle of radius a which centred at the origin has constant curvature $\frac{1}{a}$.

21. Evaluate the unit tangent vector to the graph of $r(t) = t^2 i + t^3 j$ at the point where $t = 2$.

22. Estimate an equation for the tangent plane and parametric equations for the normal line to the surface $z = x^2 y$ at the point (2, 1, 4).

23. Find the directional derivative of $f(x, y, z) = x^2 y - yz^3 + z$ at $(1, -2, 0)$ in the direction of the vector $a = 2i + j - 2k$.

24. Evaluate $f_x(1, 3)$ and $f_y(1, 3)$ by finding $f_x(x, y)$ and $f_y(x, y)$ where $f(x, y) = 2x^3 y^2 + 2y + 4x$.

25. Write the steps to find the absolute extrema of a continuous function f of two variables on a closed and bounded set R.

26. Prove that $f(x, y) = x^2 + y^2$ is differentiable at the origin.

(8 × 2 = 16 Marks)

SECTION – C

Answer any **six** questions.

27. Prove that there are $3\lfloor n/2 \rfloor$ primes in the range n through $n!$, where $n \geq 4$.

28. Let e denote the highest power of 2 that divides $n!$ and b the number of 1s in the binary representation of n. Then show that $n = e + b$.

29. Show that the gcd of the positive integers a and b is a linear combination of a and b.

30. Show that 3, 5 and 7 are the only three consecutive odd integers that are primes.

31. Find parametric equations of the tangent line to the circular helix $x = \cos t$, $y = \sin t, z = t$ where $t = t_0$, and use that result to find parametric equations for the tangent line at the point $t = \pi$.

32. Suppose that a particle moves along a circular helix in 3-space so that its position vector at time t is $r(t) = (4\cos \pi t)i + (4\sin \pi t)j + t\,k$. Find the distance travelled and the displacement of the particle during time interval $1 \le t \le 5$.

33. Find the curvature of the ellipse with vector equation $r = 2\cos t\, i + 2\sin t\, j$, $(0 \le t \le 2\pi)$ at the end points of the major and minor axes.

34. Derive Kepler's third law.

35. Find the slope of the sphere $x^2 + y^2 + z^2 = 1$ in the y- direction at the points $\left(\frac{2}{3}, \frac{1}{3}, \frac{2}{3}\right)$ and $\left(\frac{2}{3}, \frac{1}{3}, -\frac{2}{3}\right)$.

36. Verify whether the function $z = e^x \sin y + e^y \cos x$ satisfies the equation $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = 0$.

37. Locate all relative extrema and saddle points of $f(x, y) = 3x^2 - 2xy + y^2 - 8y$.

38. Use appropriate forms of the chain rule to find $\frac{\partial w}{\partial \rho}$ and $\frac{\partial w}{\partial \theta}$ where $w = x^2 + y^2 - z^2, x = \rho \sin\phi \cos\theta, y = \rho \sin\phi \sin\theta$ and $z = \rho\cos\phi$.

(6 × 4 = 24 Marks)

SECTION – D

Answer any **two** questions.

39. (a) Prove that there is no polynomial $f(n)$ with integral coefficients that will produce primes for all integers n.

    (b) Find the general solution of the LDE $6x + 8y + 12z = 10$.

40. (a) State and prove the division algorithm.

    (b) Show that the number of leap years l after 1600 and not exceeding a given year y is given by $l = \lfloor y/4 \rfloor - \lfloor y/100 \rfloor + \lfloor y/400 \rfloor - 388$.

41. A shell fired from a cannon has a muzzle speed of 800 ft/s. The barrel makes an angle of 45° with the horizontal and, for simplicity, the barrel opening is assumed to be at ground level.

    (a) Find parametric equations for the shell's trajectory.

    (b) How high does the shell rise?

    (c) How far does the shell travel horizontally?

    (d) What is the speed of the shell at its point of impact with the ground?

42. Suppose that a particle moves through 3-space so that its position vector at time t is $r(t) = ti + t^2 j + t^3 k$.

    (a) Find the scalar tangential and normal components of acceleration at time t.

    (b) Find the scalar tangential and normal components of acceleration at time $t = 1$.

    (c) Find the vector tangential and normal components of acceleration at time $t = 1$

    (d) Find the curvature of the path at the point where the particle is located at time $t = 1$

43. (a) A heat - seeking particle is located at the point (2, 3) on a flat metal plate whose temperature at a point (x, y) is $T(x, y) = 10 - 8x^2 - 2y^2$. Find an equation for the trajectory of the particle if it moves continuously in the direction of maximum temperature increase.

(b) The length, width and height of a rectangular box are measured with an error of at most 5%. Use a total differential to estimate the maximum percentage error that results in these quantities are used to calculate the diagonal of the box.

44. Find the points on the sphere $x^2 + y^2 + z^2 = 36$ that are closest to and farthest from the point $(1, 2, 2)$.

(2 × 15 = 30 Marks)