(Pages : 3) **N – 8540**

Reg. No. : ....

Name : ....

Second Year B.A. Degree Examination, August 2022

Part III: Subsidiary

MATHEMATICS

Time : 3 Hours Max. Marks 100

Half the paper carries **full** marks.

1. (a) Let $A = \{1, 2, 3\}$, $B = \{2, 3, 4\}$. Find

   (i) $A \cup B$

   (ii) $A \cap B$

   (iii) $A - B$

   (b) (i) Define supremum of a sequence and find the supremum of $x_n = 1 - \frac{1}{n}$.

   (ii) Define infimum of a sequence find the infimum of $x_n = 1 + \frac{1}{n}$.

   (iii) Find the limit of the sequence $x_n = \frac{1}{n}$.

   (c) (i) Find the derivative of $f(x) = (3x^2 + 1)(x^3 + 2x)$.

   (ii) Let x be labor and y the product that labor produces. Let us assume the relationship is $y = 3x^2 - 2x$. If we increase labor by a small amount, then how much of change will there be of the product y?

P.T.O.

2. (a) Find y' for $x^2 + y^2 = 9$.

(b) Assume that the edge of a square is measured with an absolute percentage error of at most 3%. Use a differential to estimate the absolute percentage error in computing the square's perimeter and area.

(c) Describe the concavity of $f(x) = x^3 - x$.

3. (a) Find the domain and range of the function $f(x,y) = x^2 + 2y^2 + 3$.

(b) Find all first order partial derivatives of the function $f(x,y) = x^2 - 3xy + 2y^2 - 4x + 5y - 12$.

(c) Find the n[th] derivative of $\sin 6x \cos 4x$.

4. (a) If $y = 2w^2 + 1, w = 3z^2, z = 2x + x^3$, then find the derivative of y with respect to x.

(b) Find the radius of curvature at any point of the curve $y = C\cos h(x/c)$.

(c) Is the function $f(x,y) = 4x^2 + y^2$ homogeneous? Justify your answer.

5. (a) Determine the intervals on which the function $y = 2x + x^5$ is increasing and decreasing.

(b) Find the maximum and minimum of $f(x,y,z) = 4y - 2z$ subject to the constraints. $2x - y - z = 2$ and $x^2 + y^2 = 1$ using Lagrange multipliers.

(c) Find the total differential of $w = x^3yz + xy + z + 3$ *at* $(1,2,3)$.

6. Evaluate the following:

(a) $\int \frac{x^4}{1+x^{10}} dx$

(b) $\int e^{\sin x} \cos x \, dx$

(c) $\int \frac{1+\ln x}{x}$

(d) $\int xe^x dx$

7. (a) Find the Taylor series for the function $\ln(1+x)$.

(b) Find a power series expansion of $\frac{6}{1+x^3}$.

(c) Find the sum to infinity of the series $\frac{1}{1.2}+\frac{1}{2.3}+\frac{1}{3.4}+....$

8. (a) Find the elasticity of demand at price P = 20 given the demand function Q=120 h – 4P where P stands for price and Q for quantity demanded.

(b) Find the total revenue and demand functions if the marginal revenue function is $MR = 60 - 2Q - 2Q^2$.

(c) The demand and supply curves are given by $Q_D = 66 - 3P$, $Q_s = -4 + 2P$, respectively. Find the equilibrium P and Q.

9. Solve the following:

(a) $\frac{dy}{dx}+\frac{y}{x}=x$

(b) $\frac{dy}{dx}=\frac{1-y}{1-x}$

(c) $\frac{d^2y}{dx^2}-5\frac{dy}{dx}+6y=e^{2x}$

10. (a) Solve $u_n = 2u_{n-1} - 3$, $n \geq 2$ where $u_1 = 4$

(b) Evaluate (i) Δe^x (ii) $\Delta^2 e^x$ and (iii) $\Delta \tan^{-1} x$.

(c) Solve the difference equation $y_{x+2} - 6y_{x+1} + 9y_x = 0$.

(5 × 20 = 100)